FIRST COURSE IN THE NEW MATHEMATICS

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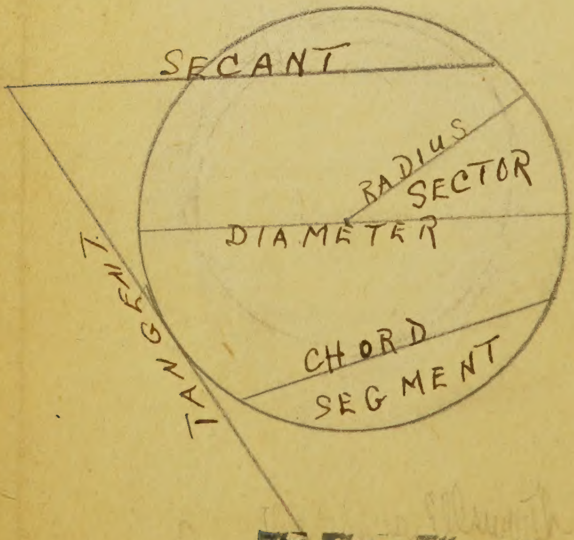

The Elmer Ellsworth
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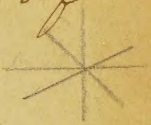
A straight line is the shortest distance between 2 points.

Thru one point any no. of lines may be drawn.

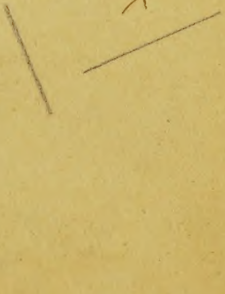


Thru 2 points only 1 straight line may be drawn.

A vertical line is an upright line.

A horizontal line is a line parallel to the horizon.

An oblique line is a slanting line.



Circle

A circle is a plane figure bounded by a curved line called a circumference every point of which is equallydistant from a point inside called the center.

A radius is a straight line from center to circumference.

A diameter is a straight line through the center to the circumference.

An arc is any part of a circumference.

A chord is a straight line joining 2 ends of an arc.

A secant is a straight line from a point outside the circle cutting a circle in 2 points

A tangent is a st. line from point outside the circle touching circle only in 1 point

Junior High School Mathematics
EDITED BY JAMES M. GLASS

A FIRST COURSE
IN
THE NEW MATHEMATICS

BY

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PREFACE

This is the first-year volume of a mathematics series for junior high schools. How it correlates, rationalizes, and applies the processes of arithmetic, geometry, and algebra is shown in the table of contents. Not only are the chapters linked in an unbroken chain of cumulative work, but within every chapter each topic develops naturally from what has gone before.

The book takes advantage of the competitive spirit of the junior high school pupil. Classroom work is socialized and made competitive in almost every section of the text. The exercises make real contacts with the pupil's life and the personal element is continually stressed even in the group work.

Special features appear in

(1) Continual emphasis on the importance of estimating.

(2) The plan for an Improvement Record Book in which the pupil constantly tests his speed and accuracy, of which he keeps a cumulative record.

(3) The treatment of graphs to meet the need for visualizing ratios. This makes the subject an integral part of the progressive development of the book.

(4) A large number of standard and other tests to help the pupil make frequent checks on his progress.

(5) Constant but varied drill through practical applications of the facts and principles learned.

(6) Simplicity and naturalness in discussion. The text talks to the child in his own language.

(7) Interest-compelling headings, section titles, and picture captions.

(8) Experimental problems which lead the pupil to discover for himself many of the major principles.

(9) Varied pictures, many from pupil life, which vitalize the subject and illustrate both principles and problems.

The problems are in no way superficial or purely theoretical; they have been thoroughly tested by the authors who have had long and practical experience in actually teaching pupils of this grade and in reviewing thousands of papers as examiners for the Regents of the State of New York.

The illustrations are more numerous than in most textbooks of this sort and form a special feature of the book. Educational psychologists are alive to the importance of attractive pictures to give interest to subjects studied by pupils of this age.

The skill tests have been prepared under the guidance of the general editor, whose experience both as a junior high school teacher and as a state director of junior high schools has made him one of the foremost experts in pupil psychology in the country.

In fine, the book aims to include and apply the seven cardinal features of this series: interest, simplicity, attractiveness, logical sequence, careful gradation, practical applications, and thoroughness.

E. I. E.
P. A. C.

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Part I aims to explain and drill the principles of the New Mathematics.

PART I: THE NEW MATHEMATICS

The book opens with estimating, which is checked by measuring. For this numbers are necessary, applied in the four fundamental processes.

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The last of these, division, leads naturally to fractions, including ratio.

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JUNIOR HIGH SCHOOL MATHEMATICS

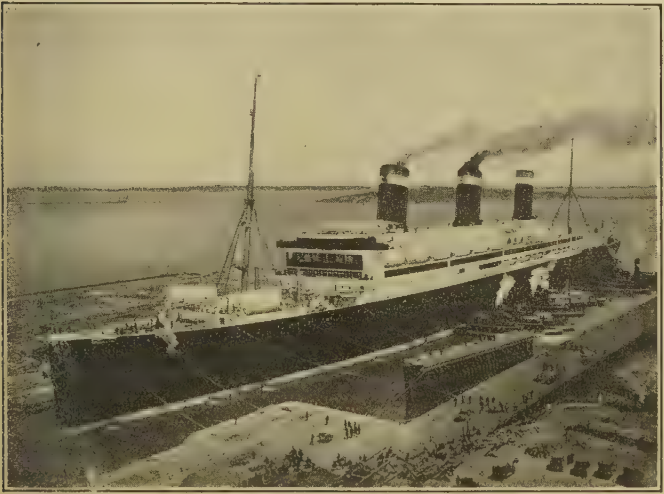
PART I

THE NEW MATHEMATICS

CHAPTER I

STRENGTHENING THE FOUNDATION. WHOLE NUMBERS

Here is a picture of the *Leviathan*, one of the largest ships afloat. How many inches wide do you think the picture is? How many inches high? Make a careful guess.



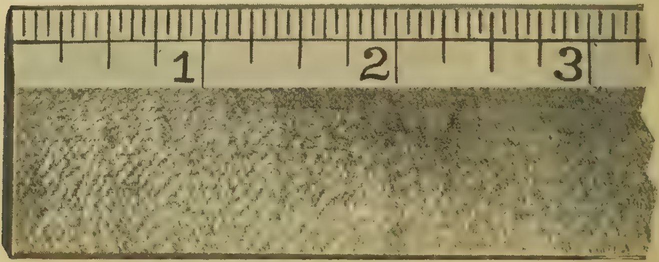
THE STEAMER LEVIATHAN

Trying to tell the size or amount of anything like the width and height of this picture is called *estimating*.

From the people, automobiles, and tugboats estimate the length of the *Leviathan*. How near is your estimate to the length as given in the index at the back of the book?

Everybody estimates, from the cook in the kitchen to the business man in the bank. If you train yourself to estimate well, you will save much time and trouble, and it will be hard to fool you with false figures. You will see at once the absurdity of your friend's statement when he says the ball field was a mile wide and there were about a million people at the game.

Measuring. — After you have made your estimate, you ought always to *check* it, that is, to find out how exact it is. To check your estimate, you must know how to measure. Ability to measure accurately is even more important than estimating and is a help in nearly everything you do.



Part of a Ruler

Here is a ruler three inches long. Take a piece of paper or cardboard and make a ruler of your own six inches long. Now measure the picture and see how well you estimated.

Turn now to the pictures on pages 4 and 6. First estimate their dimensions and then measure them. How accurate were your estimates?

In these measurements, you had to use numbers. Without a thorough knowledge of numbers you cannot measure well. If your knowledge of numbers is to be thorough, it must rest upon a firm foundation.

The Four Walls of the Foundation. — If a man builds a house one story high and later decides to add more stories, he ought first to strengthen the foundation. In your number work so far you have built up a pretty good knowledge of arithmetic. Now you are going to build upon that knowledge. Before you do so, you must strengthen the foundation.

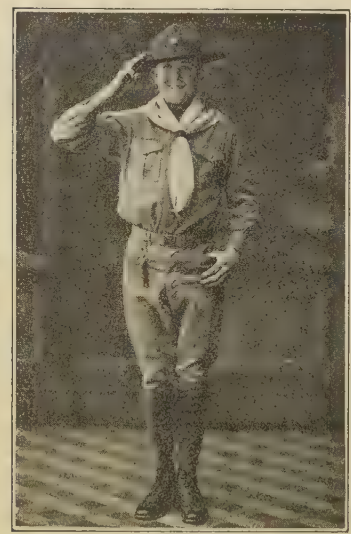
" Foundation " and " fundamental " both come from the Latin word meaning " bottom " or " base." The foundation of number work consists of the four fundamental or basic operations which are at the bottom of all mathematics. These are *addition*, *subtraction*, *multiplication*, and *division*. They may be called the four walls of the foundation, and we shall begin by strengthening them before building up our knowledge of the new mathematics.

The First Wall — Addition. — When Jack Wood went to Junior High School, he had to buy a mathematics book which cost ninety cents, an English book at a dollar, a history costing a dollar and a half, and a geography at a dollar ten. How much did his books cost all together?

Besides books he needed various other things. He bought a ruler for 15 cents, a pair of compasses for 50 cents, a protractor for 25 cents, some squared paper for 15 cents, pencils and practice paper for 35 cents, and book covers for 20 cents. How much were these articles?

How much money did Jack have to get from his father to pay for all the books and other things he needed to start in school?

When Jack told his father how much he needed, Mr. Wood found that Jack had made a mistake in adding. So he gave Jack the following sums for practice. After doing these, Jack added the list of articles without any mistake. Do these sums yourself and then add the list of books and other articles again, just to make sure that you got it right the first time.



Jack Wood
In his Boy Scout uniform.

Some Sums

Find the sums of the following:

1.	2.	3.	4.
14876	5672	723	36842
321	379	1862	7392
4821	5291	75	11406
923	86	993	729

5.	**6.**	**7.**	**8.**
53.65	76.82	87.79	564.32
12.34	28.56	26.29	125.32
9.16	17.25	341.71	38.64
237.29	461.47	35.26	9.32
12.13	8.34	7.13	12.38
5.42	74.15	16.27	6.45

Problems

1. Mr. Wood agreed to pay for Jack's lunches in the cafeteria at school if he would make a written statement at the end of each week as to how much they cost. During his first week at school they cost as follows: Monday $.24; Tuesday $.18; Wednesday $.20; Thursday $.28; Friday $.25. Read these numbers over again and figure out the total in your head as you go along. Now add them up on paper and see how much Jack's father paid him. Did you get the right answer when you worked it in your head?

2. Mr. Wood took Jack and some friends on a ten-day motor trip. The first day they drove 93 miles; the second, 147; the third, 184; and the rest as follows: 138, 169, 177, 153, 179, 162, 187. What was the total mileage for the trip?

3. The Junior High School had a school savings bank, and Jack's father told him that he would give him an allowance for his lunches if Jack would start a savings account and put something into it each month. Throughout the year Jack was able to earn money from doing odd jobs, and a large part of this he saved and put into the bank. If he deposited $2.25 in October; $2.10 in November; $.50 in December; $8.00 in January; $2.75 in February; $2.00 in March; $1.75 in April; $2.80 in May; and $3.10 in June, how much did he have in the savings bank at the end of the school year? Why was Jack's deposit in December only 50 cents?

4. James Marsden, another Junior High School boy, one month earned $2.00 for helping his mother, 60 cents for mowing a lawn, $2.00 for selling papers, and $3.15 for delivering packages. At the beginning of the month he had 15 cents in cash. How much did he have at the end of the month if he did not spend anything?

THE GIRLS' BASKET BALL TEAM

5. The Junior High School Girls Basket Ball Team was made up of Ethel Henderson, Ruth Malcolm, Mabel Johnson, Jane Stone and Polly Egan. They were great chums, especially the first three. Each girl on the team had to have one of the following outfits: 1 middy blouse $1.20; 1 pair of bloomers $2.50; 1 pair of sneakers $1.50; 1 Windsor tie $.69; 1 pair of white stockings $.50.

Find the cost of each girl's outfit.

Suppose the team wore sailor ties, which cost $3.75 each, how much more would each outfit cost?

IMPROVEMENT RECORD BOOK

When you play tennis or basket ball, you always try to improve your skill. When you run races, you keep trying for a better record. In this book you will find tests to try your skill in mathematics. In some cases the standard or average time for working the test is given; in others it is to be determined by your teacher or by a committee of your classmates. If you compare your records with these averages, you can be constantly measuring your skill.

To keep these records, you are to get a notebook, to be called your *Improvement Record Book*, which will be referred to hereafter by the initials **"I. R. B."** Write your name, school, and class on it. The tests for which records are to be entered in the **I. R. B.** are indicated by black type with a numeral in brackets, like this: **I. R. B. Test [1].**

To record your improvement, you are to take each test four times. Directions are given in the text. The following chart is suggested for your scores:

RECORDS IN I. R. B. TESTS — CHAPTER I

STANDARD TIME	TESTS	FIRST TRIAL		END-OF-CHAPTER TRIAL		REVIEW TRIAL		FOURTH TRIAL	
		Time	Accuracy	Time	Accuracy	Time	Accuracy	Time	Accuracy
	[1] Addition								
	[2] Subtraction								
	[3] Multiplication								
	[4] Division								

I. R. B. Test 1. Addition. Now take the first "Standard Test" below. In your **I. R. B.** draw a Chart similar to the one on the preceding page. Enter your time and accuracy scores in the "First Trial" column. The standard time of this test is 7 minutes. Always record this time in the Standard Time column.

Do not copy the examples; write merely the total for each column. If you wish to check your answers, add up and then add down.

347	546	231	176	258	824	735	831
546	428	374	238	481	764	569	128
612	315	452	563	392	213	312	432
154	183	175	382	182	163	181	285
472	234	543	462	582	348	263	384
671	742	845	583	674	926	284	462
123	456	789	123	456	789	123	456
273	284	915	817	716	921	832	562
145	237	358	468	924	842	751	681

Additional Addition

1. Add each of the following from left to right.

(*a*) 9 8 5 6 4 3 1 7 3 8 4 5 6 2
(*b*) 7 6 3 1 8 4 6 4 5 3 9 4 7 4
(*c*) 2 9 5 4 7 5 4 6 3 8 4 5 6 7
(*d*) 5 3 8 6 5 6 3 9 6 7 8 2 4 8
(*e*) 2 7 9 7 3 7 8 5 4 4 5 6 5 1

2. In adding long columns where there may be interruptions to your adding, write down the sum of each column separately, beginning at the right. Then add the next column and write the result one place to the left, and so on, as shown in this example:

Add:	5643	17	= total of right-hand column
	2563	24	= total of next column
	7842	24	= total of next column
	6375	29	= total of next column
	9234	31657	= total of all columns

3. Using the preceding plan, add each of the following and thus check your work.

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
5426	3476	284	52	54678
568	632	3563	4663	5281
13	25	51	653	923
2	4735	762	1	16325
57934	728	4526	6451	7484
3181	19	853	426	367
732	376	1382	1985	4216

Checking. In the subject of addition you can check either by adding up as well as down, or by putting down the sums as shown in example 2 above.

Strengthening the Second Wall — Subtraction. — Jack and his friends found that in comparing the results of tests they had to use subtraction. This seemed fun, for it was only addition reversed, and the examples were shorter. As it is simply the opposite of addition, you prove your work, or *check* it, by addition.

Simple Subtraction

Subtract, and check by addition:

1. What noted man was born in 1732 and died in 1799? How old was he when he died? How many years since his death?

2. Mr. Brown owns property valued at $75,643. He owes $56,496. How much is he actually worth?

Airplane View of New York City

3. The population of New York city in 1924 was 5,839,746. That of Chicago was 2,833,288. How much larger was the former city?

4. Give orally the difference when 9 is subtracted from each of the following:

54 67 45 98 72 36 25 23

Also subtract orally 5, 6, 7, and 8 from each number.

5. I. R. B. Test [2]. Subtraction. Standard Time 30 seconds. Enter your time and accuracy scores in your **I. R. B.** in the column of your chart marked "First Trial." Subtract the following:

15	16	9	17	7	8	18	11	19	10	12
5	7	5	9	4	1	9	6	7	4	3

14	11	6	13	16	7	10	14	12	6	8
6	3	2	5	4	5	7	5	4	4	5

6. A man had $876.25 in the bank. He drew out $456.75. How much had he left?

7. Mr. Williams left property worth $20,000 at his death. He willed $7000 to his wife, $6500 to his son, and the balance to his daughter. How much did he will to his daughter?

8. Supply the subtrahends in each of the following:

$56	$75	$86	$23	$37	$48
?	?	?	?	?	?
$26	$15	$9	$12	$10	$32

9. Subtract the second column from the first and place the result in the third column:

SELLING PRICE OF GOODS SOLD	COST OF GOODS SOLD	MARGIN (OR GROSS PROFIT)
$5000	$4000	
568	475	
87.50	65.75	
5¢	3¢	

10. Subtract thirty-six thousand eight hundred ninety-seven from fifty-nine thousand seven.

	(a)	(b)	(c)	(d)	(e)	(f)
11.	386	291	867	904	321	906
	147	145	523	677	208	711

	(a)	(b)	(c)	(d)
12.	8692	9308	14876	15921
	477	6799	7903	9428

	(a)	(b)	(c)	(d)
13.	86078302	74863324	127042650	2145163122
	30303456	43663749	73254618	605032516

(e)	(f)	(g)	(h)
240520261	350701003	372360805	640127402
101024701	262345148	168791354	367446275

	(*a*)	(*b*)	(*c*)	(*d*)
14.	$543.25	$46.86	$125.50	$98.68
	243.37	19.97	96.75	23.79

15. In subtracting 735 from 846 as follows, what is the name of 735? of 846? How do you prove an example in subtraction?

846
735

16. Subtract each of the following, and check:

6785 − 4573 =, 7328 − 6452 =, 8647 − 3754 =

Problems

1. Jack Wood decided to keep a cash account. He put it down like this:

Received					Spent				
May	1	Balance		80	May	2	Postage stamps		10
	8	Errands for neighbors		25		11	Sunday school		05
	10	Mowed two lawns		60		15	Chocolate bar		10
	15	Helped Mr. Brown		50		20	Pencil		05
	20	Mowed lawn		25		25	Sunday school		10
	25	Planted a garden	1	00		28	Ball game		25
	29	Distributed handbills	1	00			Car fare		14
							Bank Deposit for one month	2	80

What was the total amount received in May? the total spent? How much did he have left after making his bank deposit?

2. Ethel Henderson decided to keep a cash account like the one Jack Wood kept.

In the month of September she earned $.50 helping with the housework, $.40 washing dishes, $.65 running errands, and $1.25 caring for babies. She spent 15 cents for a movie, 20 cents for ice cream and candy, 10 cents for a pad, and 5 cents for a pencil. She put the balance in the school bank.

Make out a cash account similar to that found in exercise 1.

3. Ruth Malcolm went marketing for her mother one Saturday. She spent 12 cents for a package of starch, a quarter for 3 cakes of soap, 13 cents for a package of breakfast food, 30 cents for a half dozen oranges, 30 cents for 1 pound of steak, 20 cents for a pound of prunes, and 27 cents for 5 pounds of sugar. Her mother gave her a $5 bill. How much change should she bring back? How much change would she have brought back if her mother had given her only $2.75? Estimate your change as Ruth did when she made the purchases. Then check your estimate on paper.

Strengthening the Third Wall — Multiplication. — As you go on in this book, you will see that everything is planned to help you work better and faster. Multiplication is a quick way to add. You can often do it in your head.

"DO-IT-IN-YOUR-HEAD" OR MENTAL ARITHMETIC

Did you ever hear folks say, "Do-it-in-your-head"? Business men sometimes advise their clerks, "Do not waste time with pencil and paper, use your head." Time is money. Neither business men nor you can afford to waste either. Doing-it-in-your-head saves time and money.

There are many simple exercises and examples in this book. Always try first to work them in your head. You will be surprised how quickly you will grow in your ability to work mentally and how much time you will save.

GAMES AND CONTESTS IN MENTAL ARITHMETIC[1]

You and your classmates with your teacher's help can devise many games or contests in Mental Arithmetic. For instance, in example 1 of the following exercise your teacher might call for volunteers to join an Oral Contest, dividing the volunteers into two teams, and appointing a captain for each team.

Let the first team take the upper row of multiplications and the second team the lower row. Your teacher, alternating between teams, will dictate the examples. In case of a mistake, the multiplication should go to the next pupil of the opposing team; in case this pupil misses, back to the next pupil in order of the first team and so on. Each captain should score one for every correct answer of his side.

If some of your classmates are not ready to volunteer, your teacher might make a Pencil-Paper team for these pupils. This third team should work individually at their seats.

When you have finished let the Pencil-Paper pupils be divided between the two Oral teams; then all join in an oral contest of repeating the Multiplication Tables 1 to 12. Let each pupil have one trial on some one table. Score five for each whole table correctly given.

Finally, have team captains add the total scores to decide the winning team.

---

[1] The games and contests of this and all succeeding chapters are "boxed"; that is, they are surrounded by a black line. They are optional with your teacher and classmates. Your teacher will decide whether or not class time will permit their use. It may be decided to follow some suggestions, to omit others, or to devise your own plans. To gain the time needed for a class contest, you and your classmates might volunteer to do part of the lesson's written work out-of-class-hours.

Multiplication Exercise

1. Multiply each of the following by 6, writing down only the products or giving the results mentally:

49 24 71 18 52 35 94 66 83
75 34 56 27 87 92 65 48 16

If more drill is desired, use 5, 7, 8, and 9 as multipliers.

2. Multiply each of the following. You will use the first aid, addition, in multiplication. Is it the same aid as used in checking subtraction?

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)
826	947	453	861	623	563	65
32	58	75	92	37	100	1000

3. Ruth Malcolm wanted to test her skill in multiplication. She gave orally the correct products of the following to Mabel Johnson in 18 seconds. Test yourself with some other pupil and then let him try the test.

I. R. B. Test [3]. Multiplication. Enter your time and accuracy scores on your chart as in the two previous tests.

4 8 6 2 3 8 9 7 2 11
4 7 2 2 7 4 6 8 7 6

1 9 12 3 5 1 5 7 6 12
9 10 7 4 7 6 12 9 5 6

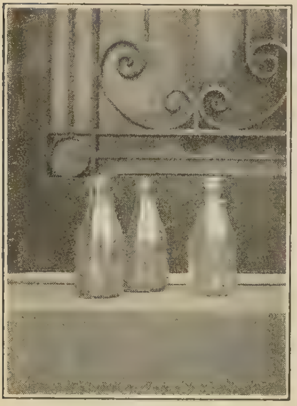
Check yourself in the following four examples by interchanging the multiplier and multiplicand and multiplying again.

4. 45 × 56. **5.** 254 × 27. **6.** 756 × 348. **7.** 354 × 98.

8. Time yourself on the following examples:

562 347 625 340 716 827
36 52 123 172 367 269

9. How many quarts of milk in 1 empty quart milk bottle? in 3 empty quart milk bottles? in 6? in 10?



THREE EMPTY BOTTLES

$1 \times 0 = ?$ $3 \times 0 = ?$
$6 \times 0 = ?$ $10 \times 0 = ?$

Any number times zero = ?

10. Multiply eighty five thousand five hundred sixty eight by twenty four thousand nine.

A MULTIPLICATION SURPRISE

Set down in a row all the digits except 8; multiply the last digit by any single digit; use this product as a multiplier of the row of digits. Now try it with the product of each digit and 9 as the multiplier. Then see if you can get all the products right; you will discover a way to check your results.

Illustrative Example. 1 2 3 4 5 6 7 9; multiply by 9 × 4.

Perform this multiplication:

$$\begin{array}{r} 12345679 \\ \underline{36} \end{array}$$

A TEAM RACE[1]

Now make a game of this work by dividing your class into teams. See which team can work the following three examples in the least time and how many correct answers each team can get. One advantage of these games is the opportunity for you and your classmates to use your own originality. Note that in this case the questions of number of teams, how to score, and the scoring of time are left for your class to decide with your teacher's help.

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[1] See footnote to page 14.

Problems

1. Mabel Johnson was elected treasurer of a Girl Scout Troop. The expenses of a Saturday trip were to be $1.20 carfare, wieners 60 cents, rolls 24 cents, marshmallows 50 cents, mustard 20 cents, and butter 20 cents. The troop decided to collect voluntary subscriptions each day for a week. On Monday, Mabel collected 10 cents apiece from 9 girls; on Tuesday, 5 cents apiece from 6 and 8 cents apiece from 4; on Wednesday, 5 girls paid 7 cents each and 7 girls paid 5 cents each; on Thursday 3 girls paid 10 cents apiece, 2 paid 15 cents each, and 6 paid 5 cents each. The voluntary subscriptions were so successful that on Friday Mabel told the Troop that they would have a good balance for another

The Girl Scout Troop

trip if each one paid 10 cents. All twelve members paid the 10 cents each on Friday. How much balance did they have for the next trip?

2. The three girls joined a Sewing Club. Ethel Henderson decided to make a kimona; she bought 4 yards of Japanese crepe at 55 cents a yard, one spool of silk for 10 cents, 9 yards of ribbon at 7 cents a yard, and 5 skeins of embroidery silk at 8 cents a skein.

Mabel Johnson bought the following goods for a house apron: 4 yards of percale at 35 cents a yard, 2 bolts of bias seam binding at 15 cents a bolt, one spool of cotton at 5 cents, and 4 buttons at 3 cents apiece.

Ruth Malcolm decided on a new dress, the goods for which cost as follows: 4 yards of voile at 58 cents a yard, 1 spool of thread at 10 cents, 8 yards of lace at 10 cents a yard, 1 card of snaps for 5 cents, and 3 yards of ribbon at 25 cents a yard. What did the materials cost each girl? Which was the most expensive article to make?

3. Ethel, Ruth, and Mabel were requested by the Principal to count the attendance at an auditorium exercise. The auditorium was divided into a right section, a center section, and a left section. The Principal told the girls that the enrollment of the school was 853. He asked them to find out how many were present. Ethel counted the attendance in the right section: 3 classes of 36 each, 2 classes of 34, 1 class of 29, and 1 of 27.

In the center section Ruth counted 2 classes of 37 each, 1 class of 39, 2 classes of 35, 1 class of 32, 1 class of 28, 2 classes of 33, and 2 classes of 34.

Mabel counted in the left section: 1 class of 35, 2 classes of 30 each, 1 class of 32, 1 class of 26, 1 class of 30, and 1 class of 33. What report should they make to their Principal?

5. Finishing the Foundation — Division. — *Division* requires the assistance of addition, subtraction, and multiplication. It is the opposite of multiplication. Thus if 9 times 7 equals 63, then 63 divided by 7 gives us 9. 63 in this example is called the *dividend*, 7 is called the *divisor*, and 9 is the *quotient*.

Again if we divide 38 by 9 we obtain 4 as a quotient with 2 left over. This is called the *remainder*. The remainder is usually written as the numerator of a fraction of which the divisor is the denominator. $38 \div 9 = 4\frac{2}{9}$.

You may check your work in division in two ways:

1. By repeating the work.

2. By multiplying the divisor by the quotient and adding the remainder, thus obtaining the dividend.

Exercise in Division

1. In the following example Ruth Malcolm made a 100% record in 30 seconds and Mabel Johnson in 28 seconds. Test yourself by these records. Do-it-in-your-head.

I. R. B. Test [4]. Division. Enter your scores as before.

$5\overline{)25}$	$10\overline{)60}$	$4\overline{)40}$	$11\overline{)66}$	$9\overline{)36}$	$9\overline{)108}$
$6\overline{)48}$	$7\overline{)35}$	$3\overline{)36}$	$12\overline{)72}$	$6\overline{)54}$	$12\overline{)96}$
$9\overline{)63}$	$8\overline{)32}$	$2\overline{)12}$	$5\overline{)40}$	$8\overline{)64}$	$11\overline{)44}$

2. Divide each of the following examples and check your work by the second method explained above:

$8\overline{)24}$	$7\overline{)28}$	$5\overline{)35}$	$8\overline{)48}$	$9\overline{)27}$	$11\overline{)99}$
$4\overline{)32}$	$5\overline{)55}$	$12\overline{)48}$	$11\overline{)55}$	$7\overline{)49}$	$6\overline{)24}$
$4\overline{)9}$	$5\overline{)11}$	$4\overline{)14}$	$6\overline{)20}$	$7\overline{)39}$	$8\overline{)50}$

3. Perform each of the following divisions:

(*a*) $25\overline{)1725}$ (*b*) $36\overline{)13500}$ (*c*) $17\overline{)7191}$ (*d*) $62\overline{)53506}$

(*e*) $46\overline{)13616}$ (*f*) $394\overline{)11032}$ (*g*) $576\overline{)36288}$

4. If an example in division does not come out even, what do you do with the remainder? For example, 17 ÷ 2 = 8 and ? $27 \div 8 = 3\frac{3}{?}$

5. Similarly divide

(*a*) 4986 by 211. (*b*) 75361 by 243. (*c*) 29841 by 167.

(*d*) 45328 by 275. (*e*) 67528 by 92. (*f*) 28757 by 177.

6. Try to divide the following 5 examples in less than 4 minutes.

$84\overline{)32000}$ $75\overline{)61500}$ $68\overline{)30600}$ $94\overline{)45000}$ $57\overline{)45087}$

Supply the missing numbers in each of the following:

7. ? ÷ 4 = 5.
8. ? ÷ 6 = 7.
9. ? ÷ 12 = 9.
10. ? ÷ 11 = 5.
11. ? ÷ 8 = 5.
12. ? ÷ 9 = 4.

13. How many packages of 5 one-dollar bills could you obtain from 45 one-dollar bills?

14. What number multiplied by 365 gives 7300?

15. How many hours will it take an automobile driver to go 150 miles if he makes a uniform speed of 25 miles an hour?

16. A gallon contains 231 cubic inches. How many gallons in a tank full of water containing 8316 cubic inches? How many gallons are there in the tank when it is half full?

17. Divide four hundred ninety seven thousand three hundred fifty two by twenty one thousand six hundred twenty four.

18. Add, subtract, and multiply each of the following sets of numbers. Then divide the upper number by the lower number. Do not use paper and pencil unless necessary.

56	45	28	36	72	54	49	32	84	35
7	9	4	12	8	6	7	8	12	7

24	39	60	56	48	25	37	81	75	66
7	3	7	9	7	6	12	8	11	9

TWO-TEAM CONTEST[1]

Plan a Team Contest for the preceding example 18. Again let your teacher divide the class into two teams and appoint a captain for each team. While your teacher dictates the examples each team member should write answers on paper. When the work is finished the team captains will collect the papers and exchange them; then distribute the opponents' papers to their own team members. While your teacher dictates the answers let each pupil check the paper distributed by the captain; then score the paper by allowing one credit for each correct addition, subtraction, multiplication, or division. Have the captains total the credits of their own teams. The greater total will determine the winning team.

Finding an Average. — Most of us know what an *average* is, though we may not be able to define it. Now we are going to use our mathematics to find averages accurately.

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[1] See footnote to page 14.

What was the average amount that Jack Wood paid for his lunch (page 5)? To find this out we add the amounts paid for the lunch for each of the five days and divide by 5.

An Average Exercise

1. Fill in the blanks in the following examples.

To find the average of 3 like quantities we add the quantities and divide by ——.

To find the average of 8 like quantities we add the quantities and divide by ——.

To find the average of *any number* of like quantities we add the quantities and divide by ——.

In the last statement written, let us use the letter n to stand for *any number*. Then we write:

"To find the average of n like numbers we add the numbers and divide by ——."

2. Jack had two written tests in mathematics one month. His teacher marked one test 82% and the other 94%. What was his *average* mark?

We solve this example as follows:

$(82 + 94) \div 2 = 176 \div 2 = 88$. His average mark was 88%.

3. At the end of the term, Mabel's standings were 92 in English, 87 in Social Studies, 85 in Science, and 96 in Mathematics. The class average in all subjects was 84. Was Mabel's average in all of her subjects higher or lower than the class average, and how much?

4. What was the average daily mileage in the trip that Mr. Wood took? See page 5.

5. When Ethel, Ruth, and Mabel counted the attendance (page 18), what was the average attendance of the twenty-five classes in the auditorium?

IMPROVING THE IMPROVEMENT RECORD BOOK

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER I

You have now strengthened the foundation of your " New Mathematics " by reviewing the four fundamental operations. While doing this you have taken four tests of your speed and accuracy and you have entered these scores in your I. R. B. under the column headed " First Trial." Now, before leaving this chapter, take the **I. R. B. Tests** again and try to improve your first scores. Keep at it until you do. Enter the time and accuracy scores of these second trials on the chart in your I. R. B. under the heading, " End-of-Chapter Trial."

CHAPTER II

BREAKING UP WHOLE NUMBERS INTO FRACTIONS FOR FINE WORK

Fractions. — Here is the finish of a quarter-mile run. There are 1760 yards in a mile. How many yards in a quarter mile?

FINISH OF A QUARTER-MILE RUN

This picture was taken in the Harvard Stadium during a Harvard-Princeton Dual Track Meet.

What part of a mile is the 220 yard dash?

A football game is divided into *halves*, and each half is again divided into two equal parts. What are these parts called?

Hockey is usually played in three twenty-minute periods. What part of the game is each period?

What part of a regular base-ball game is an inning?

Fractions are nothing new; they are a continuation of division which we have just studied. We have all used them since we first gave half our apple to a chum or a third of our marbles to each of our two brothers. The beauty of fractions is that they help us to do so many things easily, which without them would be hard.

Breaking up Materials for Finer Work

Use of Fractions. — When you motor over a highway that is being repaired you often see piles of rock and other materials gathered near a stone crusher. They are to be broken up into finer material to build the smooth road which makes driving swift and pleasant.

So in most industries large materials are broken into smaller parts for finer, quicker work.

In just this way in mathematics, the whole number one is broken into smaller parts, called *fractions*, to enable us to do finer and quicker work. Just as the skilled men, who do the finer work, are paid more than the workers on coarse raw materials, so it will pay us to learn the fine points in mathematics; this we cannot do without a knowledge of fractions.

Buying Goods

Here is some simple practice in fractions. See how many of these examples you can do-in-your-head. In buying things you should check each charge mentally.

1. Mary bought 10 pencils for a quarter. How much would one cost?

2. Henry bought 3 oranges for 10 cents. What was the cost of one?

3. John bought for his mother 4 pounds of meat at $30\frac{1}{2}$ cents a pound. How much did it cost him?

4. Everett purchased $6\frac{3}{4}$ pounds of sugar at 6 cents a pound. How much would the merchant charge him?

5. Anna's father bought $2\frac{2}{3}$ pounds of prunes at 24 cents a pound. How much did the prunes cost him?

6. How many cents will $6\frac{3}{8}$ yards of cloth cost Maud if she pays 4 dimes a yard for it?

7. One quarter dozen eggs at 6 dimes a dozen will cost —— cents.

8. Charles bought a bicycle for $36 and sold it for $\frac{3}{5}$ of what it cost him. For how much less than its cost did he sell it?

A WELL-KEPT STORE

9. Lula bought $8\frac{7}{8}$ yards of muslin at 20 cents a yard. What did it cost her?

10. State the cost of

(*a*) 72 yds. at $\$\frac{1}{3}$ a yard.
(*b*) 32 yds. at $\$\frac{5}{8}$ a yard.

11. How many yards can be bought for $1

(*a*) At $\$\frac{1}{2}$ a yard?
(*b*) At $\$\frac{1}{4}$ a yard?
(*c*) At $\$\frac{1}{3}$ a yard?
(*d*) At 10 cents a yard?
(*e*) At 20 cents a yard?
(*f*) At $2\frac{1}{2}$ cents a yard?

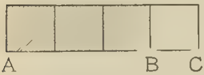
12. How many yards can be bought

(*a*) For $20 at $\$\frac{1}{2}$ a yard?
(*b*) For $4 at $\$\frac{1}{8}$ a yard?
(*c*) For $7 at $\$\frac{1}{6}$ a yard?
(*d*) For $9 at $\$\frac{1}{12}$ a yard?

Different Kinds of Fractions. — There are several kinds of fractions, all of which we must learn to recognize and name.

Common fractions have two parts, the numerator (the number above the line) and the denominator (the number below the line), called the *terms* of the fraction.

The denominator shows into how many parts the number is divided and the numerator shows how many of these parts are taken. For example, in this figure AC is a figure one inch long divided into 4 equal parts. The shaded part AB shows 3 of those 4 equal parts. Thus AB is $\frac{3}{4}$ of AC.



The fraction $\frac{3}{4}$ may also be thought of as a short way of indicating $3 \div 4$.

A *proper fraction* is a common fraction whose numerator is smaller than its denominator, as $\frac{5}{8}$, $\frac{1}{2}$, $\frac{2}{3}$.

An *improper fraction* is one whose numerator is equal to or larger than its denominator, as $\frac{5}{5}$, $\frac{9}{8}$.

A *mixed number* is a whole number (called *integer*) plus a fraction. Thus $6 + \frac{1}{2} = 6\frac{1}{2}$.

Changing the Forms of Fractions. — Fractions which have different numerators and different denominators may still be *equivalent* in value. For instance, $\frac{1}{2} = \frac{2}{4}$, for one half dollar has the same value as two quarters.

Fractions may be changed in form, but not in value, by either multiplying or dividing both numerator and denominator by the same number. The new form is still equivalent to the old.

Illustrative Example.

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}; \qquad \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6};$$

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}; \qquad \frac{4}{20} = \frac{4 \div 4}{20 \div 4} = \frac{1}{5}$$

Lowest Terms. — A fraction is said to be *reduced to its lowest terms* when both numerator and denominator are as small as they can be made without changing the value of the fraction. This is done by dividing both terms by their greatest common divisor. Always reduce fractions to their lowest terms before performing any operations. This makes your work easy. For instance, $\frac{16}{64}+\frac{18}{36}$ looks hard, but when reduced to its lowest terms it becomes easy: $\frac{1}{4}+\frac{1}{2}$.

Rapid Reductions

Reduce the following to their lowest terms. Do as many as you can in your head.

1. $\frac{6}{8}$, $\frac{12}{16}$, $\frac{9}{24}$, $\frac{21}{27}$, $\frac{18}{27}$ **3.** $\frac{14}{42}$, $\frac{27}{54}$, $\frac{28}{56}$, $\frac{24}{40}$, $\frac{18}{45}$
2. $\frac{10}{35}$, $\frac{24}{36}$, $\frac{27}{36}$, $\frac{30}{36}$, $\frac{30}{35}$ **4.** $\frac{36}{84}$, $\frac{49}{91}$, $\frac{54}{81}$, $\frac{66}{121}$, $\frac{48}{144}$

A Mixed Number is Changed to a Fraction by (1) multiplying the whole number by the denominator, (2) adding this result to the numerator, and (3) placing the total over the denominator.

Illustrative Example. $7\frac{1}{3}=\frac{(7\times 3)+1}{3}$ or $\frac{22}{3}$. If this is to be changed back to a mixed number we divide and obtain $7\frac{1}{3}$.

Even Exchanges

Change each of the following mixed numbers to improper fractions. How many can you do mentally?

1. $3\frac{1}{2}$. **3.** $8\frac{5}{6}$. **5.** $15\frac{2}{3}$. **7.** $24\frac{6}{7}$. **9.** $56\frac{1}{12}$.
2. $6\frac{3}{4}$. **4.** $5\frac{4}{5}$. **6.** $17\frac{5}{8}$. **8.** $35\frac{10}{11}$. **10.** $214\frac{1}{2}$.

Change each of the following improper fractions to whole or mixed numbers:

11. $\frac{15}{6}$. **13.** $\frac{38}{9}$. **15.** $\frac{45}{11}$. **17.** $\frac{73}{12}$. **19.** $\frac{562}{10}$.
12. $\frac{26}{4}$. **14.** $\frac{42}{5}$. **16.** $\frac{72}{7}$. **18.** $\frac{421}{8}$. **20.** $\frac{125}{25}$.

Oral Exercise

Give orally the answers to the following eight questions.

1. A dime is what part of $1? of $2? of $5?

2. 4 inches is what part of a foot?

3. 3 eggs are what part of a dozen eggs?

4. By what would you divide both numerator and denominator of $\frac{6}{9}$ to change it to $\frac{2}{3}$?

5. By what would you multiply both terms of the fraction $\frac{3}{4}$ to change it to $\frac{15}{20}$?

6. Give two fractions which are equivalent to each of the following:

$\frac{1}{4}$, $\frac{4}{6}$, $\frac{9}{12}$, $\frac{1}{10}$, $\frac{5}{100}$.

7. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{6}$ each to twelfths.

8. $\frac{7}{8} = \frac{?}{16}$

A COMPETITION AT DOING-IT-IN-YOUR-HEAD[1]

Perhaps your teacher will wish to make the following 18 examples a do-it-in-your-head competition. Have half the class stand in line. Let pupil 1 work example 1, and so on. The other half of the class check the answers at their seats. Then vice versa. In case there are not enough examples your teacher will furnish more, similar to those given here.

When the class has finished the examples orally, each pupil should write the answers. Have your teacher select three pupils who made 100% records in the oral work to decide with her on a standard time for working the 18 examples. These three pupils and your teacher will be the *Steering Committee* to direct contests and to set standard times for I. R. B. tests.

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[1] These games and competitions which are boxed in a black line are optional. Your teacher will decide whether time will permit their use or whether they are adaptable to your class work. See footnote to page 14.

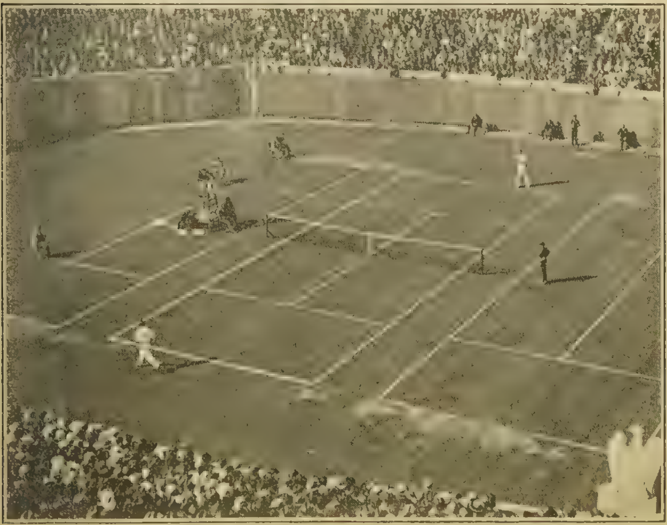
Practice

Reduce to lowest terms and read the results of the following 15 examples:

1. $\frac{8}{10}$. **4.** $\frac{63}{81}$. **7.** $\frac{30}{35}$. **10.** $\frac{15}{20}$. **13.** $\frac{48}{60}$.
2. $\frac{12}{18}$. **5.** $\frac{4}{12}$. **8.** $\frac{14}{28}$. **11.** $\frac{26}{39}$. **14.** $\frac{16}{64}$.
3. $\frac{7}{21}$. **6.** $\frac{16}{24}$. **9.** $\frac{22}{33}$. **12.** $\frac{35}{49}$. **15.** $\frac{34}{51}$.

16. How many thirds are equal to eight twelfths?

17. Mr. Williams raised 400 bushels of apples, of which 50 bushels were inferior in quality. What part was inferior?



A BIG TENNIS MATCH

This picture shows Johnston (at this side) and Tilden (opposite), two great American players, on the courts at Forest Hills, New York.

18. A tennis player won 7 matches out of the 8 that he played. What part did he win? What part did he lose?

I. R. B. Test [1]. Fractions. Prepare a chart for four tests in your I. R. B. similar to the one used for your scores in Chapter I. Do the following 12 examples in your head. Write your time and accuracy scores as in the tests in Chapter I. Let the Steering Committee set a standard time.

1. One third is equal to how many sixths?

2. One fourth is equal to how many eighths?

3. Change to twelfths: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{6}$.

4. Change to sixths: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{12}$, $\frac{12}{12}$, $\frac{6}{9}$.

Fill in the correct numbers in the place of the question mark in each of the following:

5. $\frac{2}{3} = \frac{?}{12}$. **7.** $\frac{1}{2} = \frac{?}{8}$. **9.** $\frac{5}{12} = \frac{?}{36}$. **11.** $\frac{3}{?} = \frac{6}{8}$.

6. $\frac{5}{6} = \frac{?}{12}$. **8.** $\frac{5}{8} = \frac{?}{16}$. **10.** $\frac{6}{8} = \frac{?}{4}$. **12.** $\frac{5}{6} = \frac{60}{?}$.

Addition and Subtraction of Fractions. — To add or subtract fractions first change each fraction to an *equivalent* fraction in such a way that the resulting fractions will have a *common denominator*. We cannot add $\frac{1}{2}$ and $\frac{1}{3}$ until we change each of them to sixths, $\frac{3}{6}$ and $\frac{2}{6}$. It is sometimes best to change mixed numbers to improper fractions before adding or subtracting.

Illustrative Example.

$$\frac{3}{4} + \frac{5}{6} = \frac{9 + 10}{12} = \frac{19}{12} = 1\frac{7}{12}$$

Exercise

1. Which is greater, $\frac{1}{2}$ or $\frac{1}{3}$? How much greater?

2. What is the sum of $\frac{1}{3}$ and $\frac{1}{4}$?

3. To add or subtract fractions what must we first find?

Add the following:

4. $\frac{1}{2}+\frac{2}{3}$.

5. $\frac{3}{4}+\frac{1}{2}$.

6. $\frac{2}{3}+\frac{3}{2}$.

7. $\frac{3}{5}+\frac{1}{2}+\frac{1}{3}$.

8. $\frac{2}{3}+\frac{1}{2}+\frac{1}{5}$.

9. $\frac{1}{5}+\frac{1}{7}+\frac{3}{10}$.

10. $\frac{2}{3}+\frac{1}{2}+\frac{4}{5}$.

11. $\frac{4}{7}+\frac{1}{2}+\frac{3}{8}$.

12. $5+\frac{1}{3}+\frac{3}{5}$.

13. $2\frac{1}{2}+3+5\frac{1}{3}$.

14. $3\frac{1}{2}+2\frac{2}{3}$.

15. $6\frac{3}{4}+2\frac{1}{2}+4\frac{1}{4}$.

Perform the following operations:

16. $\frac{2}{3}-\frac{1}{2}$.

17. $\frac{3}{5}-\frac{1}{3}$.

18. $\frac{5}{6}-\frac{1}{4}$.

19. $\frac{5}{8}-\frac{1}{4}$.

20. $\frac{3}{4}-\frac{1}{3}$.

21. $\frac{5}{6}-\frac{3}{4}$.

22. $\frac{5}{7}-\frac{1}{2}$.

23. $\frac{5}{12}-\frac{1}{3}$.

24. $\frac{7}{12}-\frac{2}{5}$.

25. $\frac{15}{16}+\frac{3}{4}-\frac{7}{12}$.

26. $3\frac{1}{2}+4\frac{3}{4}-1\frac{5}{6}$.

27. $2\frac{8}{9}-1\frac{5}{8}+2\frac{5}{6}$.

Multiplying Fractions. — (1) *To multiply a fraction by a whole number (integer): Divide the denominator by the integer if the result will be a whole number; if not, multiply the numerator by the integer.*

(2) *To multiply two or more fractions, first divide any numerator and any denominator by the largest number which is contained in both.* (This is sometimes called *canceling.*) *Then multiply all numerators for a new numerator and all denominators for a new denominator. If the canceling has been properly done, the answer will be in its lowest terms.*

Illustrative Examples.

1. $\cancel{6}\times\frac{5}{\underset{3}{\cancel{18}}}=\frac{5}{3}=1\frac{2}{3}$.

2. $7\times\frac{5}{8}=\frac{35}{8}=4\frac{3}{8}$.

3. $\frac{\cancel{3}}{\cancel{4}}\times\frac{\overset{2}{\cancel{8}}}{\underset{\cancel{3}}{\cancel{9}}}\times\frac{\overset{4}{\cancel{12}}}{19}=\frac{8}{19}$.

4. To multiply a mixed number by an integer, for example

$$\begin{array}{rl} 14\frac{1}{2} & \\ 9 & \\ \hline 4\frac{1}{2} & = 9\times\frac{1}{2} \\ 126 & = 9\times 14. \\ \hline 130\frac{1}{2}. & \textit{Ans.} \end{array}$$

5. To multiply a mixed number by a mixed number.

$$7\tfrac{1}{2} \times 12\tfrac{1}{3} = ?$$

$$7\tfrac{1}{2} = \tfrac{15}{2}$$

$$12\tfrac{1}{3} = \tfrac{37}{3}$$

$$\frac{\overset{5}{\cancel{15}}}{2} \times \frac{37}{\underset{1}{\cancel{3}}} = \frac{185}{2} = 92\tfrac{1}{2}. \quad \textit{Ans.}$$

$$\begin{array}{rl}
24\frac{1}{2} & \\
36\frac{1}{3} & \\
\frac{1}{6} & = \frac{1}{3} \times \frac{1}{2} \\
8 & = \frac{1}{3} \times 24 \\
18 & = \frac{1}{2} \times 36 \\
144 & = 6 \times 24 \\
72 & = 3 \times 24 \\
\hline
890\frac{1}{6}. & \textit{Ans.}
\end{array}$$

FINDING PRODUCTS

Common Sense Can Convince You

Estimating is of great value in multiplication. It means applying your common sense. For instance, in example 1 on the next page, if you estimate that $2\frac{1}{2} \times 14\frac{1}{4}$ is 28, your common sense tells you that your estimate is absurd because not only is 2×14 equal to 28 but also $\frac{1}{2}$ of 14 is 7, which added to 28 is 35; certainly then your answer is over 35.

Estimates are for just this purpose — to prevent absurd answers.

In the following table, one answer is absurd and one is correct. First estimate, then check the correct answers as rapidly as you can.

1.	$2\frac{1}{2} \times 14\frac{1}{2}$	28	$36\frac{1}{4}$		**6.**	$\frac{3}{4} \times 816$	612		400	
2.	$\frac{2}{3} \times 400$	$266\frac{2}{3}$	300		**7.**	$\frac{3}{4} \times \frac{3}{4}$	$1\frac{1}{2}$		$\frac{9}{16}$	
3.	$100 \times \frac{5}{8}$	$62\frac{1}{2}$	75		**8.**	$21\frac{1}{3} \times 6\frac{2}{3}$	$142\frac{2}{9}$		126	
4.	$\frac{2}{3} \times \frac{5}{6}$	$\frac{1}{4}$	$\frac{5}{9}$		**9.**	$\frac{3}{4} \times 250$	750		$187\frac{1}{2}$	
5.	$5 \times 2\frac{2}{3}$	$13\frac{1}{3}$	$10\frac{1}{2}$		**10.**	$25 \times 2\frac{1}{2}$	50		$62\frac{1}{2}$	

TWO HEADS ARE BETTER THAN ONE

Mathematics "comes naturally," as the saying is, to some of you, but to some others it "comes hard." This gives your class a fine opportunity for some good team work.

Those of you who are getting pretty good marks can help boost the class average by first getting your own work out of the way and then helping out some classmates who need help. Such team work will produce splendid results.

You can plan your own way of working together. Perhaps your teacher will make the assignments of couples to work together and will let you know which pair of pupils gets the best results from the scheme. Then the rest of the class can, in future work of the same sort, follow the plan of the winning couple. If the teacher wishes, you may try the following test by couples.

I. R. B. Test [2]. Find the products of the following, reducing answers to lowest terms. Keep records of time and accuracy and enter them in I. R. B., First Trial column. Let the Steering Committee set a standard time.

1. $4 \times \frac{5}{9}$.
2. $3 \times \frac{7}{12}$.
3. $6 \times \frac{3}{11}$.
4. $7 \times \frac{9}{14}$.
5. $\frac{5}{2} \times 16$.
6. $\frac{1}{2} \times \frac{1}{3}$.
7. $\frac{3}{2} \times \frac{4}{9}$.
8. $\frac{3}{4} \times \frac{2}{3}$.
9. $\frac{5}{8} \times \frac{2}{15}$.
10. $8 \times 16\frac{1}{2}$.
11. $15\frac{1}{5} \times 25$.
12. $6\frac{2}{3} \times 3$.
13. $5\frac{1}{2} \times 4\frac{1}{3}$.
14. $\frac{3}{5}$ of $\frac{5}{10}$.
15. $\frac{3}{10}$ of $\frac{2}{27}$.

Aliquot Parts. — An aliquot part is a part which divides a whole number evenly. Thus 10, $12\frac{1}{2}$, $16\frac{2}{3}$, 20, 25, $33\frac{1}{3}$, and 50 are all aliquot parts of 100, for they divide it evenly into 10, 8, 6, 5, 4, 3, and 2 parts, respectively; that is, they are respectively $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$, of 100.

It is often convenient to use these fractions instead of the corresponding aliquot parts. This is shown in the following:

Illustrative Example. — What will 16 pounds of sugar cost at $12\frac{1}{2}$ cents a pound?

Solution: Since one pound costs $12\frac{1}{2}$ cents, or $\frac{1}{8}$ of \$1, 16 pounds would cost 16 × \$$\frac{1}{8}$, or \$2.

See how much easier it is to work the example this way than to multiply 16 × $12\frac{1}{2}$ cents.

Do-it-in-your-head

Give the cost of:

1. 60 dozen eggs at 50 cents a dozen.
2. 24 pounds potatoes at $12\frac{1}{2}$ cents a pound.
3. 15 pounds pork at $33\frac{1}{3}$ cents a pound.
4. 12 pounds of stew meat at $16\frac{2}{3}$ cents a pound.
5. 20 gross of toothpicks at 25 cents a gross.
6. 36 yards of cloth at $33\frac{1}{3}$ cents a yard.
7. 25 quarts of berries at 20 cents a quart.
8. 48 heads of lettuce at $12\frac{1}{2}$ cents a head.

Fractions in Everyday Use

1. Jack Wood worked $18\frac{1}{2}$ hours for Mr. Berry and received 38 cents an hour. How much did he earn?

2. A cubic foot of water contains $7\frac{1}{2}$ gallons. How many gallons in a tank containing $72\frac{3}{4}$ cubic feet of water?

3. Mr. Marsden bought $12\frac{3}{4}$ tons of coal at $\$13\frac{1}{2}$ a ton. What was the cost?

4. What will 5 pieces of cloth cost at $8\frac{1}{2}$ cents a yard if each piece is 30 yards long?

5. The cost of $18\frac{1}{2}$ yards of ribbon at $\$1\frac{1}{2}$ a yard = ?

6. A firm sold $32,640 worth of goods in one year. Their margin (or gross profit) was $\frac{1}{4}$ of their sales. What was their margin of profit in money?

7. I bought $6\frac{1}{2}$ yards of velvet at $\$5\frac{1}{2}$ a yard. If I offered two $20 bills in payment, how much change should I receive?

8. It is $8\frac{3}{4}$ miles from town A to town B, $6\frac{1}{4}$ miles from B to C, $5\frac{5}{8}$ miles from C to D. Find the cost of building a concrete road from A through B and C to D at $60,000 a mile.

Dividing Fractions. — Do you recall the four walls of the foundation of arithmetic? Give them in order. Which was used as an aid to check subtraction? How did you check multiplication? How many aids were used in division? How did you check division?

You will use these aids in just the same way in fractions as you did in whole numbers. Why are they called the *four fundamental operations?* If you have forgotten, look back on page 3.

16 bushels ÷ 4 = ? 16 fifths ÷ 16 = ? 12 pints ÷ 6 = ?
16 cents ÷ 8 = ? 9 dimes ÷ 3 = ? 15 eights ÷ 5 = ?
$\frac{16}{17} \div 2 = ?$ $\frac{5}{3} \div \frac{2}{3} = \frac{5}{3} \times \frac{3}{2} = ?$ $\frac{6}{11} \div 5 = \frac{6}{11} \div \frac{5}{1} = \frac{6}{11} \times \frac{1}{5} = ?$

Can you state a rule for the last example? Ruth Malcolm tried to state it and said "To divide one fraction by another we . . . the divisor and then multiply." She could not recall the omitted word. What is it? What does it mean?

I. R. B. Test [3]. When you have finished the examples below on paper try them again mentally (in-your-head). Keep the records of your oral time and accuracy in your I. R. B. First Trial column. Let the Steering Committee set a standard time.

Divide each of the following:

1. 5 cents ÷ 5.
2. 6 halves ÷ 3.
3. 8 fourths ÷ 4.
4. $\frac{9}{8} \div 9$.
5. $\frac{7}{2} \div 4$.
6. $\frac{5}{8} \div 3$.
7. $\frac{6}{7} \div \frac{3}{7}$.
8. $\frac{7}{24} \div \frac{7}{12}$.
9. $\frac{9}{10} \div \frac{7}{8}$.
10. $\frac{4}{5} \div \frac{3}{4}$.
11. $\frac{15}{16} \div \frac{3}{4}$.
12. $\frac{51}{64} \div \frac{17}{32}$.
13. $\frac{39}{76} \div \frac{26}{57}$.
14. $\frac{75}{87} \div \frac{15}{58}$.
15. $\frac{63}{119} \div \frac{14}{51}$.

Division of Mixed Numbers. — (1) In dividing a mixed number by an integer or another mixed number, change them all to improper fractions, invert the divisor, and multiply.

Illustrative Example 1. Divide $18\frac{1}{2}$ by 7.

Solution: $18\frac{1}{2} = \frac{37}{2}$.
$\frac{37}{2} \div 7 = \frac{37}{2} \times \frac{1}{7} = \frac{37}{14}$.

(2) When the divisor is an integer which will go evenly into the parts of the mixed number, use the following method:

Illustrative Example 2. Divide $6\frac{3}{8}$ by 3.

Solution: $6 \div 3 = 2$. $\frac{3}{8} \div 3 = \frac{1}{8}$.
Therefore $6\frac{3}{8} \div 3 = 2\frac{1}{8}$.

(3) If two larger mixed numbers are divided you will find the following method easier than to reduce each to fractions.

Illustrative Example 3. Divide $362\frac{5}{8}$ by $16\frac{1}{2}$.

Solution:

$$\begin{array}{rrl} 16\frac{1}{2} & 362\frac{5}{8} & \\ 8 & 8 & \\ \cline{2-2} 132 &)2901(21\frac{43}{44} & \\ & 264 & \\ \cline{2-2} & 261 & \\ & 132 & \frac{129}{132} = \frac{43}{44} \\ \cline{2-2} & 129 & \end{array}$$

Explanation. We must first find the lowest common denominator of 8 and 2, which is 8. Then we multiply each of the mixed numbers by 8, thus changing them to eighths. Then divide as shown at the left.

Division Drill

Before solving the following examples, examine each carefully and choose the easiest of the three methods for working.

1. $12\frac{3}{4} \div 3$.
2. $27\frac{9}{15} \div 9$.
3. $10\frac{2}{15} \div 8$.
4. $256\frac{2}{3} \div 2\frac{1}{2}$.
5. $66\frac{2}{3} \div 5$.
6. $36\frac{3}{8} \div 3$.
7. $48\frac{5}{9} \div 6$.
8. $56\frac{4}{7} \div 14\frac{1}{6}$.
9. $191\frac{3}{4} \div 4\frac{1}{7}$.
10. $144 \div 16\frac{2}{3}$.
11. $87\frac{1}{2} \div 12\frac{1}{2}$.
12. $321 \div 7\frac{3}{5}$.
13. $561\frac{7}{8} \div 25\frac{1}{2}$.
14. $256\frac{3}{4} \div 38\frac{7}{8}$.
15. $487\frac{5}{9} \div 78\frac{1}{3}$.

I. R. B. Test [4]. In the following examples first estimate the answers. Use your common sense and put down only what you think are *sensible estimates*. Doubtful estimates are of no value. Then work each example and see how near your estimates are.

Record the time and accuracy scores of your estimates in I. R. B., First Trial column. Let the Steering Committee set a Standard Time. Count only exact answers in your accuracy score.

1. $5\frac{5}{8} \div 5$.
2. $12\frac{3}{4} \div 3$.
3. $15\frac{3}{7} \div 3$.
4. $16\frac{1}{2} \div 2$.
5. $7\frac{1}{7} \div 5$.
6. $8\frac{3}{4} \div 4$.
7. $17 \div 4\frac{1}{4}$.
8. $111 \div 12\frac{1}{3}$.
9. $8 \div 2\frac{2}{3}$.
10. $19\frac{2}{3} \div 59$.
11. $6\frac{1}{2} \div 3\frac{1}{4}$.
12. $9\frac{3}{4} \div 4\frac{1}{2}$.
13. $64\frac{5}{8} \div 6\frac{3}{4}$.
14. $8\frac{2}{3} \div 1\frac{7}{12}$.
15. $17\frac{3}{4} \div 2\frac{1}{2}$.

16. By what would you multiply 18 to equal $14\frac{2}{5}$?

17. How many times 9 is 10 times $2\frac{3}{20}$?

18. In the olden days 8 barrels of flour cost $\$33\frac{3}{5}$. What did one barrel cost then? 20 barrels?

19. If $3\frac{1}{2}$ pounds of flour now costs 28 cents, what is the price of 1 pound? of 1 barrel of 196 pounds?

20. Mr. Malcolm drove his automobile 175 miles in $6\frac{1}{4}$ hours. What was his speed per hour?

REVIEW TRIAL OF I. R. B. TESTS FOR CHAPTER I

Do not take the class time for this exercise. Try to improve your scores all by yourself. Try again the four I. R. B. tests in Chapter I, which you have already recorded in your I. R. B. On the page where you entered your scores for the first and second trials make a record of this third trial. If you are not satisfied, try again until you have improved your scores.

Do not think that you must always be watched in taking these tests. A Junior High School pupil is certainly old enough to be trusted. If you falsify your records, you cheat only yourself. The honorable and sensible thing is to go after a better record, as you would do in athletics.

Remember also that anything which brings discredit upon you brings even greater disgrace to your class and to your school. Neither life in-school nor life out-of-school has a place for a cheat. So after all, one great test which you are making in these I. R. B. Tests is the trial of your honor. You will be a worthy Junior High School boy or girl if you never fail to pass this test with a perfect record.

COACHING COUPLES AND COACHING COMMITTEE[1]
OR
"C. C. AND C. C. GAME"

Before starting the next two exercises, "Recipe Problems" and "Store Problems," each pupil should work independently six original examples in fractions which your teacher places on the blackboard. Give each pupil $16\frac{2}{3}$ credits for each correct answer. Let the three pupils who first secure 100% records be a Coaching Committee to assist the teacher.

Divide the rest of the class evenly according to their scores. Let each pupil in the high-score half of the class work with some one from the low-score half who wants his help; each such pair of pupils will compose a Coaching Couple.

The high-score pupil of each Coaching Couple should let the low-score pupil work the examples of the following two exercises, assisting only when necessary. If any couple needs assistance, the Coaching Committee and your teacher will help.

In "C. C. and C. C. Games" your class may decide to take one class period, or a part of one, for the work of the game. Then, after getting this start, the Coaching Couples may agree upon a time and place, either in school or at home, to complete the work.

Recipe Problems from the Cooking Class

In their cooking class Ruth Malcolm and Mabel Johnson planned the following problems from their recipes. The examples prove that they were good students as well as good cooks.

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[1] See footnotes to pages 14 and 30. These "boxed" games may always be omitted and the following examples done as a regular exercise.

PUTTING RECIPES TO WORK

1. A certain recipe for molasses cookies calls for $1\frac{1}{2}$ cups of brown sugar. It is a recipe for 3 dozen cookies. Suppose that you wanted to make 9 dozen cookies for a camping party, how much sugar would you require?

2. A recipe for a pound of fudge calls for $\frac{1}{2}$ cup of milk. If you were to make $\frac{1}{2}$ pound of fudge, how much milk would you require?

3. It requires $4\frac{1}{2}$ quarts of flour in a recipe to make six loaves of bread. How much would it require to make 18 loaves of bread?

4. If $\frac{1}{4}$ cup of flour will make 4 griddle cakes, how much flour will be required to make enough cakes for 12 persons, allowing 6 cakes to a person? for 4 persons?

5. A certain cake recipe for 6 people calls for $1\frac{1}{2}$ cups of sugar and $\frac{2}{3}$ of a cup of butter. How much of each would you need to make up a recipe for 3 people? for 8 people?

6. In a recipe for Spanish cream for 8 people, $\frac{1}{4}$ box of gelatine is called for. How much would you use in a recipe for 4 people? for 12 people?

7. A recipe for cottage pudding to serve 12 people uses ¼ cupful of butter, ⅔ cup of sugar, 1 egg, 1 cup of milk, 2¼ cups of flour, 4 teaspoons baking powder, and ½ teaspoon of salt. Adapt this recipe for 6 people; for 8 people; for 9 people.

Fractions of All Kinds in Store Problems

James Marsden and Ethel Henderson visited several stores to find store problems. They selected the following from a long list which they secured. The search for the store problems by one couple will suggest to you problems that might best be solved by the "C. C. and C. C." game.

1. One Saturday James Marsden bought his mother a 3½ pound roast at 38 cents a pound. What did it cost him?

2. How many cents will 4⅔ yards of cloth cost Ruth if one yard cost 6 dimes?

3. In former days Jack Wood's father bought some meat at 5 cents a pound and paid $3⅖ for it. How many pounds did he buy?

4. In those days how much would 13¾ yards of cotton cloth cost at 13 cents a yard?

5. In a store in Baltimore one winter day Mabel Johnson bought ½ dozen eggs at 72 cents a dozen. What did they cost her?

6. Find the total cost of 6 pounds of sugar at 6½¢ a pound, 2½ quarts of milk at 12¢ a quart, 1½ pounds of butter at 54¢ a pound, ½ pound of lard at 34¢ a pound, and ½ pound of coffee at 58¢ a pound. How much change should I receive from a $5 bill?

7. State the number of pounds that can be bought for

$16 at $½ a pound.
$15 at $⅓ a pound.
$24 at $¾ a pound.
$36 at $⅔ a pound.

8. Certain shirts were selling at $36 a dozen. If $\frac{1}{3}$ of the selling price is the margin (or gross profit), how much margin was the merchant making on a dozen? on 1 shirt?

9. Mrs. Malcolm bought $7\frac{1}{2}$ yards of cloth at $8\frac{1}{2}$¢ a yard. How much change should she receive from a half dollar and a quarter?

10. A merchant was selling a certain article for $4.50. On account of poor business he marked it down $\frac{1}{3}$. What was his new marked price on this article?

11. Mrs. Henderson spent $\frac{3}{4}$ of a dollar buying rice at $\frac{1}{8}$ of a dollar a pound. How many pounds did she get?

12. A fruit dealer was selling bananas at 39 cents a dozen. He told Ethel that he would give her one if she could tell him how many would cost $22\frac{3}{4}$ cents. Can you tell?

A BOY SCOUT GARDENING PROJECT

Edward Towner was a member of a Boy Scout Troop in his Junior High School. The Scoutmaster told the troop that each boy who could make money by helping some one else would receive a merit badge. Edward's father, who was a farmer and gardener, learned of this offer and decided to give Edward a plot of ground for a garden.

1. Edward's father told him that he might have all his profits for his bank account. Edward decided to set out strawberry plants on one half of his garden and devote the rest to early vegetables. One third of this latter portion of the garden was planted to peas, one half to onions, and the rest to lettuce. What part of the garden was planted to peas? $\frac{1}{3} \times \frac{1}{2} = ?$ Does $\frac{1}{3}$ of $\frac{1}{2}$ mean the same as $\frac{1}{3} \times \frac{1}{2}$? What part of the garden was planted to onions? $\frac{1}{2} \times \frac{1}{2} = ?$ See if you can find exactly what part of the garden was planted to lettuce?

2. Strawberries do not begin to bear until the second year. At the end of that time he sold 156 quarts of berries at an average price of 20 cents a quart, 320 heads of lettuce at 5 cents each, 210 bunches of young onions at 10 cents a bunch, and $4\frac{1}{2}$ bushels (How many quarts in a bushel?) of green peas at 20 cents a quart. What was his income from his garden in these two years?

3. Edward's expenses for the two years were as follows: For purchasing tools \$5; for fertilizer \$6 each year; for lime \$2; for strawberry plants \$8; for seed 60 cents. What amount did he make for his bank account?

Courtesy of Boy Scouts of America

EDWARD AND SOME OF HIS TROOP

Mr. Towner was so well pleased with Edward's success in his garden project that he said: "You are now two years older and able to help me in all the farm work. If you will give, during this third year, your out-of-school and summer time to the work about the farm, I will pay you 10 cents on each dollar the farm earns." Edward accepted the offer.

4. Mr. Towner's farm contains 440 acres, half of which is woodland. How many acres are woods? 60 acres are pasture and he keeps 30 cows. How many acres does each cow have to use?

5. Last year Mr. Towner sowed 40 acres to wheat, which yielded 35 bushels to the acre; 10 acres to oats, which yielded 50 bushels to the acre; and planted 12 acres to potatoes, which yielded 160 bushels to the acre. How many bushels of grain did he raise? The remainder of the farm was in hay and he cut on the average 1 ton of hay to the acre. How many tons of hay did he cut? What part of his farm was in hay?

6. Mr. Towner valued his wheat at $1 a bushel; his oats at 80 cents a bushel; his potatoes at 60 cents a bushel; and his hay at $20 a ton. What was the total value of these crops? Which crop yielded the greatest value per acre?

7. He reserved 60 tons of hay, 100 bushels of wheat, and 150 bushels of oats to feed his cattle, horses, and fowls. He sold the rest at the prices quoted in example 6. What was his cash income from these crops? What was Edward's share at 10 cents of each dollar's value?

8. Edward had deposited his garden earnings in the bank and now deposited also his share of the farm income. What was his total bank deposit?

Extra Practice with Fractions

1. $12\frac{1}{2} + 18\frac{3}{4} + 27\frac{1}{4} + 42\frac{5}{8} = ?$
2. $5\frac{7}{10} + 2\frac{5}{12} + 4\frac{2}{15} + 8\frac{4}{5} = ?$
3. $214\frac{1}{2} + 315\frac{3}{4} - 296\frac{7}{8} = ?$
4. $(7\frac{1}{2} - 2\frac{3}{4}) + (6\frac{1}{3} - 2\frac{5}{6}) = ?$
5. $(12\frac{3}{4} + 3\frac{1}{2}) - (5\frac{1}{6} + 6\frac{2}{3}) = ?$
6. $3\frac{1}{2} \times 7\frac{1}{4} \times 4\frac{3}{4} = ?$
7. $32 \times 8\frac{3}{16} \times 7\frac{1}{2} = ?$
8. $\frac{3}{7}$ of $\frac{8}{9}$ of $\frac{14}{15}$ of $\frac{5}{16} = ?$
9. $35\frac{1}{3} \div 2\frac{1}{4} = ?$
10. $15\frac{1}{2} \div 32\frac{1}{4} = ?$

Review

1. There are 3846 people watching a ball game. One-sixth of the number are boys. How many boys are there?

2. From $\frac{3}{4}$ of $\frac{2}{3}$ take $\frac{1}{2}$ of $\frac{3}{4}$.

3. At \$$1\frac{2}{5}$ each how many books can I buy for \$$37\frac{4}{5}$?

4. $16\frac{1}{2}$ yards + $21\frac{3}{4}$ yards − $7\frac{7}{8}$ yards − $19\frac{7}{16}$ yards = ?

5. Susan bought $5\frac{1}{2}$ yards of cloth and used $3\frac{3}{4}$ yards. How many yards had she left?

6. A man runs at the rate of $9\frac{23}{33}$ yards per second. How far will he run in $10\frac{3}{5}$ seconds?

7. If grapefruit are sold at 2 for a quarter, how many will \$$1\frac{3}{4}$ buy?

8. What is the cost of 10 eggs at 65 cents a dozen?

9. A man made \$250 on an investment of \$2000. What fractional part of his investment did he make?

10. If a man runs his automobile $18\frac{1}{2}$ miles on a gallon of gasoline, how many gallons will it take at the same rate to run $277\frac{1}{2}$ miles?

11. A lady took a trip of 125 miles on two railroads. If she traveled 30 miles on railroad A and the balance on railroad B, what fractional part of the total cost of the trip should each receive?

12. A man receives an annual income of \$2400. If he spends $\frac{2}{5}$ of it for food, $\frac{1}{6}$ for clothing, and saves $\frac{1}{2}$ of the remainder, how much does he save?

13. After spending $\frac{5}{8}$ of her money a lady had \$60 left. How much had she at first?

14. $56\frac{3}{4} + 27\frac{1}{2} + 15\frac{2}{3} + 31\frac{3}{4} + 66\frac{2}{3} + 72\frac{2}{5} = ?$

15. If a liter of water weighs $2\frac{1}{5}$ pounds, how many liters in 60 pounds of water?

16. A tank contained 476 gallons after it had been $\frac{1}{3}$ emptied. How many gallons would it contain when full?

17. $3\frac{4}{5} + 7\frac{1}{2} \times 3\frac{1}{4} - 5\frac{1}{2} \div 2 = ?$

18. Which is the longer run and how much, 13 times around a quarter mile track or 7 times around a half mile track?

19. If one bushel occupies $1\frac{1}{4}$ cubic feet, how many bushels in a box containing 1000 cubic feet?

20. A man spent $\frac{1}{3}$ of his money for one article and $\frac{1}{2}$ of his money for another article. If he had $600 left, how much had he at first?

21. One rouble (Russian money) is approximately $$\frac{1}{2}$. How many roubles in $$25\frac{1}{2}$?

22. One shilling (English money) is approximately $$\frac{1}{4}$. How many dollars in 125 shillings?

23. If $\frac{5}{8}$ of a number is 425, what is $\frac{3}{4}$ of the number?

24. In a mile college relay race the first boy ran his quarter in $53\frac{1}{2}$ seconds, the second in $52\frac{1}{2}$ seconds, the third in $54\frac{3}{4}$ seconds, and the last or pivot boy in $50\frac{1}{2}$ seconds. How long did it take the team to run the race?

25. If $4\frac{3}{4}$ yards are required for an article, how many articles could be made from 76 yards of cloth?

END-OF-CHAPTER TRIAL OF I. R. B. TESTS FOR CHAPTER II

In this chapter you have seen whole numbers broken up into fractions for finer work. You have reviewed different kinds of fractions and have learned that the four fundamental processes are used with fractions as well as with whole numbers.

You have also learned coöperation (working together) as illustrated in the " C. C. and C. C." games. Team work always makes work better and more enjoyable.

Then, too, you have been given a chance to make a clean record of honor and trustworthiness. You alone know whether you passed that test with a 100% record. If *you know* that you did, never permit that record to be lowered.

Now turn back through the pages of Chapter II and find the tests for which you entered records in your Improvement Record Book. Have your class decide how you will take the second trials of these four tests, whether in class, in school, or at home.

Compare your present End-of-Chapter records with the First Trial records to see if you are improving the Improvement Record Book. Enter your new time and accuracy scores in the column, End-of-Chapter Trial.

CHAPTER III

FRACTIONS AS RATIOS PICTURED IN GRAPHS

In the last chapter we thought of a fraction as a division to indicate part of a number. We are now going to study fractions as *ratios* to indicate the relation of two numbers, that is, a comparison between them.

The Brown Family

John is sitting next to the nurse.

Fractions as Number Relations. — Mary Brown is 12 years old. She has two younger brothers. John is 4 years of age and Horace is 9 years old. Mr. Wood asked Mary how her age compared with John's and she said, "I am three times as old as John." "Then," said Mr. Wood, "John is a third as old as you are." Mary then thought that she would express the relation of her age to Horace's in a similar manner, and said: "I am $\frac{4}{3}$ as old as Horace, for $\frac{4}{3}$ of 9 is 12." "Right," said Mr. Wood, "that means Horace is $\frac{3}{4}$ as old as you."

Ratio. — A *ratio* is the quotient of two *like* numbers. This quotient may be expressed in several ways but is most frequently written as a fraction, usually reduced to its lowest terms. To find the ratio of one number to another, divide the first by the second.

The ratio of Mary's age to John's age is three to one, $\frac{3}{1}$ or 3.

The ratio of John's age to Mary's age is one to three, $\frac{1}{3}$ (read "1 is to 3").

The ratio of Mary's age to Horace's is four to three, $\frac{4}{3}$ (read "4 is to 3").

The ratio of Horace's age to Mary's is three to four, $\frac{3}{4}$ (read "3 is to 4").

The ratio of Horace's age to John's age is ?

The ratio of John's age to Horace's age is ?

Rapid Ratio Work

1. Remembering that ratios, when represented by fractions, must be reduced to lowest terms, give the ratio of

6 to 8
5 to 15
3 to 12
2 to 10
6 to 30
3 to 15
8 to 4
18 to 6
40 to 5
8 to 12
4 to 6
12 to 15
30 to 20
14 to 7
28 to 21

2. What is the ratio of

3 cents to 5 cents?
4 bushels to 12 bushels?
5 dollars to 25 dollars?
9 dimes to 3 dimes?
12 yards to 72 yards?
1 inch to 1 foot?
5 quarts to 3 bushels?
3 inches to 1 yard?
4 dimes to 2 dollars?
5 nickels to one dollar?

Hint: As ratios express relations between *like* numbers only, feet must be changed to inches, bushels to quarts, and so on, in the right hand column above.

3. What number has the same ratio to 12 that 4 has to 6?

4. What number has the same ratio to 100 that 15 has to 20?

5. What number has the same ratio to 8 that 30 has to 20?

6. Mary baked a cake. The recipe called for 4 cups of flour and 1½ cups of milk. Mary wanted a smaller cake, so she used only 3 cups of flour. How much milk should she use?

AN ATTRACTIVE LIBRARY

7. In a certain town subscriptions were taken for a library. A rich man offered to give $2 for every $5 given by the other citizens. They subscribed $24,360. How much did the rich man give?

8. What is the ratio of one week to one day? of one day to one hour?

9. What is the ratio of 3 days and 6 hours to 2 days and 4 hours? *Hint:* Change each to hours.

10. What is the ratio of 1 yard and 2 feet to 10 yards?

11. What is the ratio of 4 gallons and 2 quarts to 3 gallons?

12. What is the ratio of 1 meter to 1 kilometer?

13. What is the ratio of $\frac{1}{2}$ pound to 5 pounds and 8 ounces?

14. Draw a line on your paper 3 inches long, and under it another line 2 inches long. Then draw a third line $2\frac{1}{2}$ inches long. What is the ratio of the first line to the second line? of the second line to the first line? of the third line to the first? (There are 6 half-inches in the first line and 5 half-inches in the third.) What is the ratio of the first line to the third? of the third line to the second? of the second line to the third?

15. Here is a line **a** ________________.
Draw a line **b** if the ratio of **b** to **a** is $\frac{1}{2}$.
Draw a line **c** if the ratio of **c** to **a** is $\frac{2}{3}$.
Draw a line **d** if the ratio of **d** to **a** is $\frac{3}{4}$.
Draw a line **f** if the ratio of **f** to **a** is $\frac{3}{2}$.
Draw these lines first by estimating the proper lengths with your eye. Then measure the line **a** and draw them accurately. Which member of the class estimated most accurately?

Pictures of Ratios. — Diagrams showing the relations (ratios) of numbers are called *graphs*. They are a great help in making these relations clear. There are many different kinds of graphs, but at present we are going to study only some of the simpler ones.

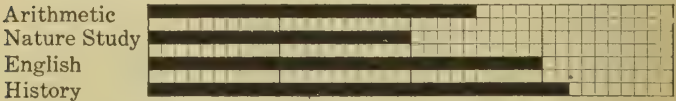
For making graphs you will need graph paper, often called "squared paper." You will find some in the pocket on the back cover of this book, but for your work in graphs you must get several large sheets.

Notice that there are squares of two sizes. How many small squares long is one large square? The side of a small square is what part of the side of a large square? If the side of a large square is called 1, what would the side of a small square be called?

There are many different kinds of graph paper, but we shall use only two in this book. One has 100 small squares in each large square; the other has only 25. We shall generally use the former.

Bar-Graph Pictures. — The first graph we are going to study is called a *bar graph.* It is illustrated by the following example.

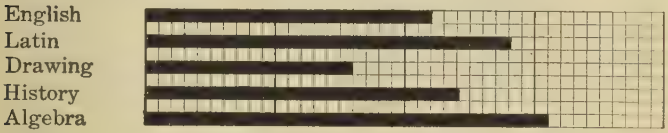
In Mary Thompson's class in arithmetic there are 25 boys and girls. In her class in nature study there are 20. In her class in English there are 30, and in her history class there are 32. The comparative size of these classes is shown by these heavy black lines called *bars*. These show the ratio of each class to every other class.



Graph of Mary Thompson's Classes

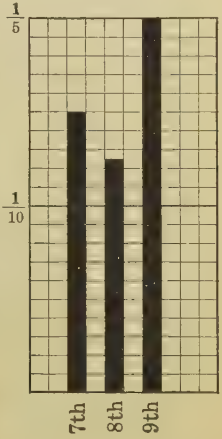
A bar exactly as long as one large square represents 10. How many pupils does a bar as long as 1 small square represent? The first bar is as long as $2\frac{1}{2}$ large squares, and thus represents $2\frac{1}{2} \times 10$, or 25 pupils. The second represents 20, the third 30, and the fourth 32.

Below are bars representing the number of pupils in each of Helen Freeman's classes. The squares represent the same units as in the graph of Mary Thompson's classes. What is her largest class? her smallest? See if you can tell the exact number in each class.



Graph of Helen Freeman's Classes

Sometimes bar graphs are made vertical. The next one shows the part, or ratio, of the pupils who failed and could not be promoted in the seventh, eighth, and ninth grades of the Memorial Junior High School. Look at the graph and tell which grade was most successful. Tell as accurately as you can what part of each grade failed. There were 240 pupils in the seventh grade, 184 in the eighth, and 160 in the ninth. How many pupils in that school failed of promotion in each of these grades?

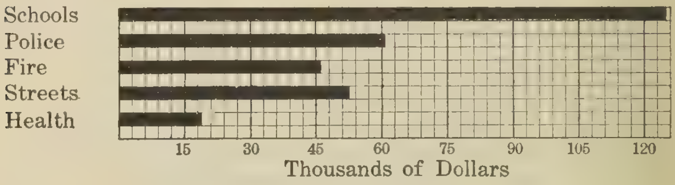


Graph of Failures

Making Bar Graphs. — Bar graphs are very useful to represent large numbers. In such cases they are not supposed to be absolutely accurate.

Illustrative Example. In a certain city the cost of the public schools was \$125,350; police \$60,100; fire department \$45,800; streets and parks \$52,600; public health

$18,250. Picture the relation of these costs by the use of bar graphs.



Graph of City Expenses

Solution: Draw a heavy vertical line near the left side of your graph sheet, and make all the bars start evenly at that line. The longest bar must represent $125,350. The longest that we can make it with the paper that we are using is 9 large squares. Each large square then must represent at least $\frac{1}{9}$ of $125,350, which is nearly $14,000. Let us then make one large square represent $15,000 and then each small square here will represent $3000 if we use paper like that above that has 25 small squares in one large square. $125,350 ÷ 15,000 = 8 large squares, with $5350 remaining. $5350 ÷ 3000 = 1.78$^+$ small squares.

Hence the bar, which represents $125,350, has a length equal to 8 large squares and a little more than $1\frac{3}{4}$ small squares. In a similar way we divide $60,100 by 15,000 to find the bar representing police expenses; and so on for the other departments.

Bar-Graph Pictures of Number Relations

1. Construct bars to show the ratio of the number of boys to girls in your grade.

2. If you can obtain the information, construct bars to represent the number of pupils in three different grades in your school.

3. The distance by the New York Central Railroad
From New York to Poughkeepsie is 75 miles.
From New York to Albany is 143 miles.
From New York to Syracuse is 291 miles.

Draw three bars to represent these distances.

What is the ratio of your second bar to the first one?

The third one is about —— times as long as the first one.

The second one is about —— times as long as the third one.

If you prefer, select distances between your home town and three or more other well-known towns.

4. In a certain school there were 35 pupils in the sixth grade, 30 pupils in the seventh grade, and 25 in the eighth. $\frac{1}{5}$ of the sixth grade, $\frac{1}{6}$ of the seventh grade, and $\frac{12}{100}$ of the eighth grade failed to be promoted at the end of the year. Make bars to represent the number of pupils in each grade which were promoted. *Hint:* Use vertical bars.

5. The Woolworth building in New York City is 792 feet high, the Metropolitan Life building is 700 feet high, the Singer building is 612 feet high, and the Municipal building is 560 feet high. Construct vertical bars to show these relations.

Municipal Building
New York City.

6. In a small class in the Woodrow Wilson School the following were the marks in a term examination: Mary 93;

John 78; Susan 80; Henry 94; William 82; Catherine 77; Everett 90; William 75, and Lloyd 95. Make bar graphs to show the relation of these marks.

7. The number of deaths due to automobiles in New York state was:

1906	66	1921	1629
1914	568	1922	2029
1920	1410	1925	2120

Make a bar chart to show the above facts.

What is the approximate ratio of the bar for 1922 to the bar for 1914?

The bar for 1925 is about how many times as long as that for 1906?

8. The number of automobiles and trucks registered in New York state was:

1906	11,753	1921	779,451
1914	169,966	1922	966,928
1920	649,672	1925	1,637,670

Make bars to represent these numbers approximately.

What is the ratio of the bar for 1922 to that for 1914?

The bar for 1925 is about how many times as long as the bar for 1906?

If you can secure the figures for your own state, use them instead of the above figures in making your graph.

9. The deposits each year in the savings banks of the United States are reported as follows:

1900	2,449,547,885 dollars	1924	8,439,855,000 dollars
1910	4,070,486,246 dollars	1925	9,065,181,000 dollars
1920	6,536,470,000 dollars	1926	9,599,118,000 dollars

Show these relations by the use of bar graphs.

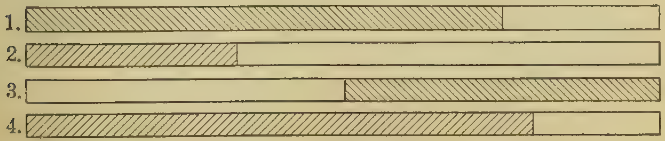
10. Obtain at the office the number of absences in your school for four successive months. Then make bar graphs to represent these figures.

11. Bring to class any bar graphs which you may find in magazines or papers and explain them to the class.

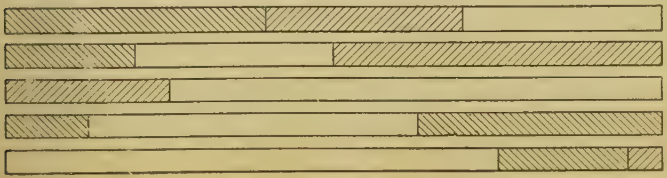
12. If you have a savings bank in your school, try to obtain the savings for each class per month and make bar graphs to represent them.

Ratios Pictured by the Rectangle Graph. — Sometimes rectangles are useful to show the relation of parts to the whole. They are constructed with some convenient length representing the whole. 10 large squares (or 5 if the paper is short) often work best to represent the whole, for they contain 100 small squares.

Tell which of the following rectangles is about $\frac{3}{4}$ shaded, which $\frac{1}{2}$ and which $\frac{1}{3}$ shaded? Which is about $\frac{1}{5}$ unshaded?



Which of the following rectangles are divided in ratio $\frac{1}{4}$ and $\frac{3}{4}$? $\frac{2}{5}$, $\frac{3}{10}$, and $\frac{3}{10}$? $\frac{3}{4}$, $\frac{1}{5}$, and $\frac{1}{20}$? $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{1}{8}$? $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{3}{10}$?



Rectangle Graphs which Illustrate Ratios

1. Construct a rectangle 24 small squares long and 2 small squares wide to represent the 24 hours of a day, and in it show the following relations applying to a pupil in Junior High School.

School	6 hours	Play and work	4 hours
Sleep	9 hours	Other uses of time	4 hours
Study	1 hour		

2. From your chart in example 1, estimate the ratio of the time spent in play to that spent in school. The time spent in school to that for sleep. The time for sleep to that for study. The time for other things to that in school.

3. Mr. Slocum had a yearly income of $3000. If he should use $\frac{1}{4}$ of it for food, $\frac{1}{5}$ for rent, $\frac{3}{20}$ for fuel and light, $\frac{3}{20}$ for clothing, $\frac{1}{10}$ for carfare, $\frac{1}{10}$ for sundries, $\frac{1}{20}$ for doctor and dentist, find the amount of money that he could apply to each of the above. Construct a rectangle chart and divide it up into the proper parts to show these relations. *Hint:* Use 5 large squares to represent $3000, then 1 large square will represent how much? 1 small square?

4. Find the total automobile deaths from the following for a certain year.

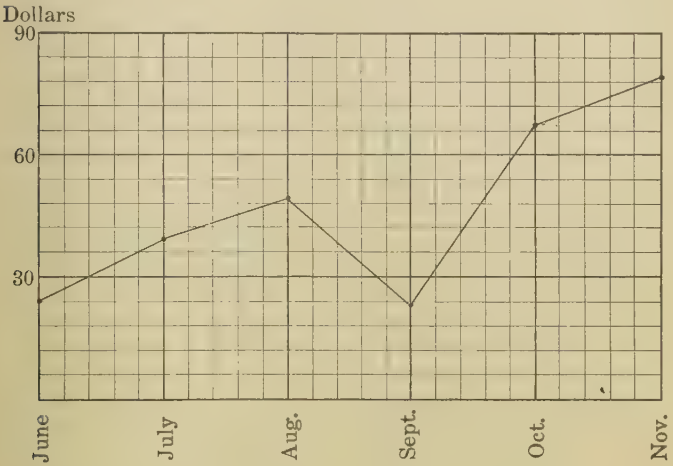
New York	945	Detroit	257	Boston	132
Chicago	583	Los Angeles	237	Rochester	47
Philadelphia	304	St. Louis	166	Jersey City	35

By letting 1 square inch or 1 large square represent 500, draw a rectangle graph to show the total of the above and then divide it into the proper parts to represent the deaths in each city.

Pictures of Ratios by the Broken-Line Graph. — Another kind of graph which is very often used is the broken-line graph.

Illustrative Example.

In June, Jerome Thomas had $24 in the savings bank. In July he had $39, in August $50. Then he bought a suit of clothes that cost $27, so that he had only $23 in the bank in September. The following month his uncle gave him $50 for his birthday. He spent $5 of this and put $45 in the bank, so that in October he had $68 in the bank. In November he had $80. Make a graph showing the amounts that he had in the bank each month.



Graph of Jerome Thomas's Savings

Solution:

Draw a heavy black line down the left side of the paper. Across the bottom draw another which meets the first line so as to make an exactly square corner. These lines are called *axes*, and the point where they meet is called the *origin*. Mark this point 0. Then on the horizontal (see dictionary) axis as shown in the above diagram mark off the months in order on the heavy lines as shown here, beginning with the vertical axis under which we write June. On the vertical axis we mark off amounts of money, letting each large square stand for $30. Then of course each small square will represent $\frac{1}{5}$ of $30, or $6. (Here we are using 1 large square divided into 25 small squares.)

Now Jerome had $24 in the bank in June, so we mark the point above June at a position to represent $24. (It may help you to think of this point as the corner of $24 Street and June Avenue.) This would be 4 small squares up for $24. In the same manner on the line marked July (or you can think of it as July Avenue) we locate and mark the point to represent $39, and so on for the other months. When these points are all located and marked, we connect them with a broken line, as shown in the diagram above.

This graph gives us a fine idea of the way in which Jerome's bank account grew.

Do you think that he ought to try to keep the line always going upward? Why?

If it were your bank account, would you want the broken line to be steep or more nearly horizontal? Explain why?

Broken-Line Pictures of Number Relations

Construct broken-line graphs to represent the following:

1. Jack Wood's earnings for a year were:

Jan. . .	$3	May . .	$3.50	Sept. . .	$6
Feb. . .	2	June . .	4.50	Oct. . .	5
Mar. . .	2.50	July . .	25	Nov. . .	4
Apr. . .	4	Aug. . .	20	Dec. . .	3.50

Can you guess why the line might go up so high in July and August?

2. Jack's spendings were:

Jan. . .	$1	May . .	$1	Sept. . .	$5
Feb. . .	.50	June . .	2.50	Oct. . .	1
Mar. . .	.75	July . .	0	Nov. . .	.50
Apr. . .	.50	Aug. . .	.50	Dec. . .	5

Can you guess why

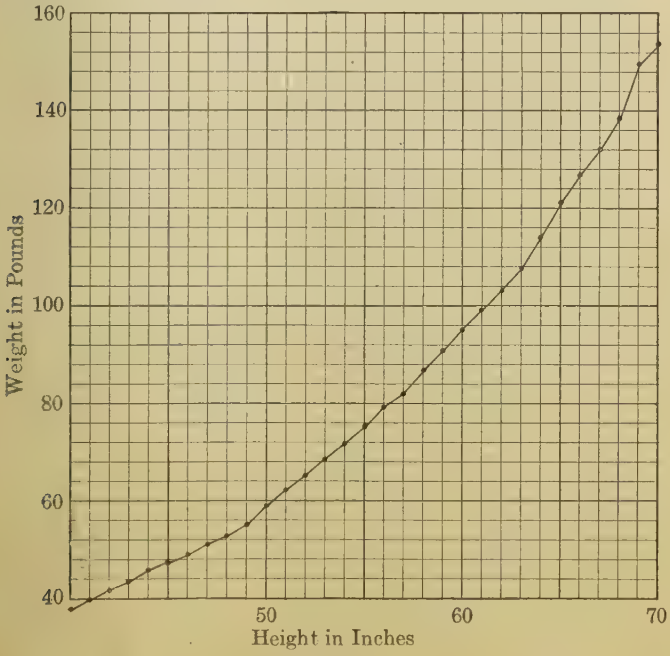
Jack's spendings were so much in June, September, and December?

Jack's spendings were nothing in July?

3. Find out how much Jack saved in each of the above months. Then try to graph his savings with a broken line. *Hint:* Watch out in December. Can you suggest how to arrange the problem for that time?

4. James Marsden received the following marks in his mathematics work for the year:

Sept. .	.75	Dec. .	.85	Mar. .	.65	June .	.80
Oct. .	.90	Jan. .	.78	Apr. .	.75		
Nov. .	.81	Feb. .	.70	May .	.87		

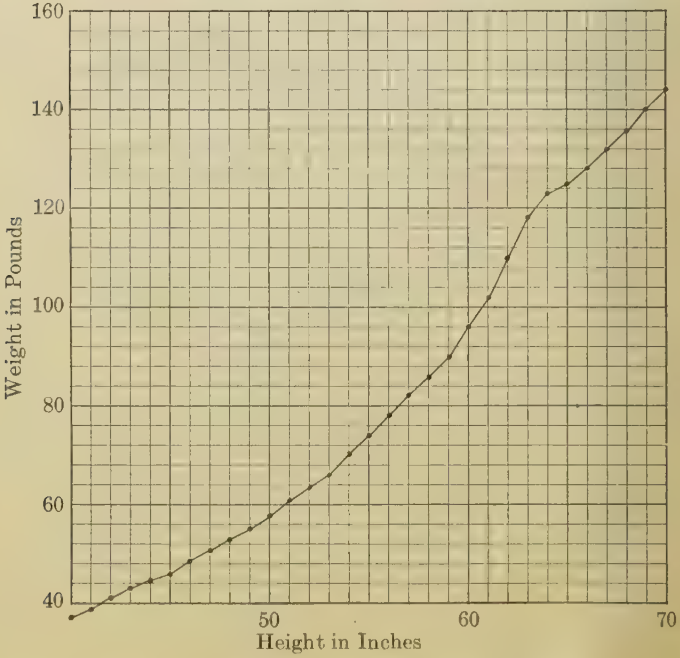


Graph 1. Standard Weight and Height of Boys

Make a broken-line graph of these marks. In which month is the graph highest? lowest? Find the average of his marks for the year.

5. I. R. B. Record. Keep your report-card standings in different subjects and make broken-line graphs of them. Arrange the graphs so as to show your school marks for successive months; for example, let vertical lines represent standings, horizontal lines represent months, and different kinds of broken lines, like — — — —, — ·· — ·· — ··, ······ or different colored inks, etc., represent the different subjects. Try it and see how well you like this new kind of report card. Keep this graph in your I. R. B.

6. Graph 1 on preceding page is a graph showing the



Graph 2. Standard Weight and Height of Girls

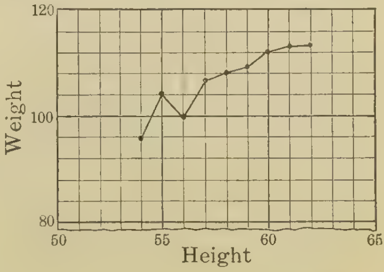
standard weight of boys for heights from 40 inches to 70 inches.

Graph 2 on the preceding page gives similar information for girls. Compare these two graphs.

7. I. R. B. Record. Let each pupil of the class construct such a graph showing the relation between his own height and weight early in his Junior High School course. Put this record in your I. R. B. Later in the year make another similar graph.

In case the weight is markedly below that indicated by the standard graphs, and especially if the graph turns downward for no apparent reason, consult your physician.

8. Here is a graph of the weight and height of Anthony Simmons recorded for the three years of his Junior High School course. How much "overweight" was he at the beginning of the course? How much at the end? Early in his course he had a severe attack of influenza. How is this indicated in the graph? Do you think that he recovered satisfactorily? Why?



Graphic Exercise

Graphs are pictured mathematics to make ratios clearer.

1. The monthly report cards of the seventh-grade class in mathematics showed class averages as follows: Sept. 86, Oct. 83, Nov. 92, Dec. 90, Jan. 94. Draw a graph to represent these averages. Do you think that this was an excellent class?

2. In the same class the averages in English were Sept. 88, Oct. 91, Nov. 92, Dec. 85, and Jan. 93. Make a graph of these averages.

3. Draw the graph again to show both the mathematics and English averages on the same squared paper, using red for the mathematics and blue for the English.

4. Edward Towner's report card contained the following standings: Sept. Mathematics 81, English 85; Oct. Mathematics 86, English 80; Nov. Mathematics 94, English 94; Dec. Mathematics 90, English 88; Jan. Mathematics 90, English 85. Edward wanted to compare his standings with the class averages. He drew two graphs, one in mathematics and a second in English, using black for the class average and red for his own. Draw the graphs.

5. A health instructor asked the boys of one class to compete with the girls of another class in a record of daily tooth brushing. There were 30 in each class. The contest ran for two weeks. The records were kept on one graph card in the gymnasium. Make a similar graph from the following records:

First week: Monday 15 boys brushed their teeth; Tuesday 20 boys; Wednesday 25 boys; Thursday 28 boys; Friday 30 boys.

Monday 20 girls; Tues. 20 girls; Wed. 25 girls; Thur. 29 girls; Fri. 30 girls.

Second week:

The boys' record was 30, 29, 27, 25, 28.

The girls' record was 26, 26, 30, 28, 30.

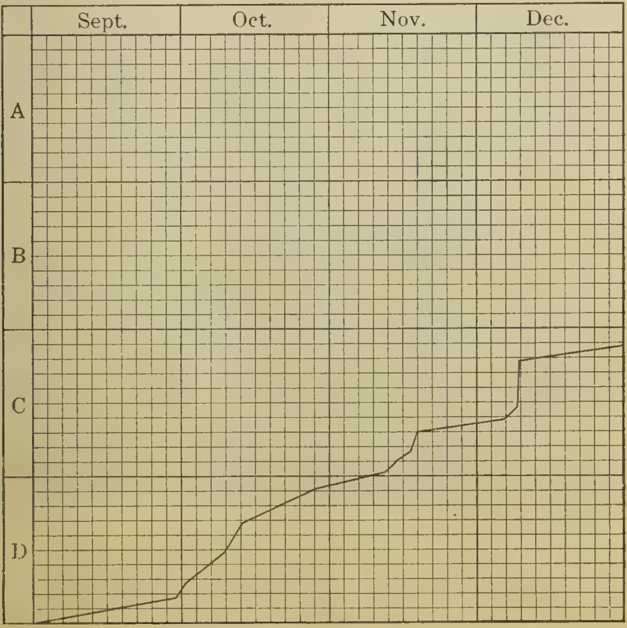
6. The approximate death rate per 1000 persons in an Atlantic seaboard city is given below.

1910 .	. 16	1914 .	. 14.6	1918 .	. 17.9	1922 .	. 11.9
1911 .	. 15.5	1915 .	. 14.6	1919 .	. 13.4	1923 .	. 11.7
1912 .	. 14.7	1916 .	. 14.6	1920 .	. 12.9	1924 .	. 11.6
1913 .	. 14.6	1917 .	. 14.6	1921 .	. 11.2		

Construct a broken-line graph of the above.
What might you think happened in that city in 1918?
Which year had the lowest death rate?

7. Try to obtain similar information from your city health department or the town clerk and graph those facts for your town or city.

8. The graph below from the John Muir Junior High School, Los Angeles, California, shows how corrective health work will remedy underweight. See if you can explain it.



Graph of Gain in Posture Rating

Graphs and Ratio in Estimating. — Graphs are a great help in estimating. They help us see things at a glance. The ratio of the estimate to the exact answer should be as close as possible. Business men are constantly using graphs and estimating.

Mr. Hawley, a merchant, gave this problem to his three sons, Alfred, Clarence, and Fred: "A man bought $8\frac{1}{2}$ yards of cloth for $3.75. How much could he get for $15?"

Alfred obtained as an answer 43 yards. Clarence obtained 30 yards and Fred 25. Mr. Hawley did not wish to take the trouble to work this out with pencil and paper, so he made a mental estimate like this: "$3.75 is a little less than $4, and $15 is a little less than four times 4, so for $15 one could get about four times $8\frac{1}{2}$ yards, or about 34." He concluded that Clarence was nearest the right answer. Do you agree?

On another occasion the boys tried to divide 36.72 by 6.8. This time Alfred obtained 3.6, Clarence 5.4, and Fred 7.8. Mr. Hawley saw at once that $36 \div 7 = 5\frac{1}{7}$. Since the divisor is a little less than 7, the quotient would be a little more than $5\frac{1}{7}$, therefore he decided that Clarence was right again.

A GRAPH GAME[1]

Have a committee appointed from your class to gather facts which can be pictured by bar, rectangle, or broken-line graphs. Appoint two members of the class to act with your teacher as judges. Ask members of the class to prepare graphs of the facts gathered and present these graphs to the judges. Let the winner be known as the "Champion Graph Artist" and let him keep a graph record on the blackboard of the class attendance for a week.

If there is more than one class studying graphs, a second contest may be held between the champions.

Everyday Estimating

1. The girls of Ethel's class conducted a candy sale one Friday afternoon for the benefit of the Near East Relief. Ethel was appointed to purchase the necessary supplies. The class contributed $4.50 for the purpose. Ethel said that she would buy 50 pounds of sugar, 10 cakes of chocolate, 15 cans of condensed milk, and 2 pounds of butter. Sugar cost 7 cents a pound, chocolate 20 cents a cake, milk 15 cents a can, and butter 55 cents a pound. Jack Wood laughed at her when she told him about it, and asked her what she was going to do with the rest of the money. Why do you think he laughed?

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[1] All suggested games are optional.

THE BOYS IN THE LUNCH ROOM

2. Max one day bought at the school lunch room a ham sandwich for 7 cents, a bottle of milk for 5 cents, a roll for 2 cents, a pat of butter for 1 cent, and a piece of pie for 7 cents. Arthur was cashier. Max gave him 50 cents and Arthur handed back 22 cents. Max said he was sure that the lunch cost less than a quarter, and he ought to have more than a quarter left. Was he right?

In each of the following, first estimate, then work, then check.

3. $5\frac{1}{2}$ tons of coal at $12 a ton.

4. 465 barrels of flour at $3.50 a barrel.

5. $2\frac{1}{4}$ pounds of meat at 40¢ a pound will cost ——.

6. $12\frac{1}{3}$ days of work at $4.80 a day amounts to ——.

7. $8\frac{1}{2}$ pounds of sugar at $6\frac{1}{2}$¢ a pound is ——.

8. What is the weight of 31 cubic feet of water if 1 cubic foot weighs $62\frac{1}{2}$ pounds?

A Review Check on Estimates

I. R. B. Test [1]. First Trial Scores. Prepare a chart for Chapter III (3 tests) similar to the ones you drew in your I. R. B. for Chapters I and II. In each test let the Steering Committee and teacher fix a time for the class. First estimate, then solve accurately, each of the following 16 examples. Record the time and accuracy scores of your estimates as you did in Test [4] of Chapter II. Remember that any estimate which is counted in your accuracy score should be an accurate answer. Your score will probably be unsatisfactory but you can improve it in your next trials.

Addition.

1. 36 + 30 + 25 + 36 + 10.

2. 260 + 350 + 110 + 290.

3. $16.80 + $5.20 + $11.75 + $62.15.

4. $3\frac{1}{2}$ yds. + 5 yds. + 8 yds. + $2\frac{1}{2}$ yds. + 6 yds.

Subtraction.

5.	**6.**	**7.**
1875	$625.10	3125
320	180.45	1810

8. John bought a pair of shoes for $7.50, a tie for $1.50, a muffler for $2.50, and six collars at 30 cents each. How much change should he receive from a $20 bill?

Multiplication.

9. Find the cost of 20 grapefruit at "two for a quarter."

10. $\begin{array}{r}625\\ \underline{16}\end{array}$ **11.** $\begin{array}{r}\$312.50\\ \underline{165}\end{array}$ **12.** $\begin{array}{r}2186\\ \underline{12}\end{array}$

Division.

13. $250 \div 12\frac{1}{2}$.

14. \$186520 ÷ 20.

15. $6532 \div 40 = ?$

16. $52960 \div 150 = ?$

REVIEW TRIAL OF I. R. B. TESTS, CHAPTER II

Try again the four I. R. B. Tests of Chapter II. On the chart where you entered the First and End-of-Chapter scores record your Review Trial scores in the proper column. Read again the directions given in the Review Trial of Chapter I, page 40.

Miscellaneous Review

1. See if you can state the results correctly in 20 seconds or less in the following additions:

$$\begin{array}{ccccccccccccccc}3&7&4&1&2&8&1&5&5&4&3&8&2&2&5\\ \underline{7}&\underline{8}&\underline{8}&\underline{4}&\underline{6}&\underline{8}&\underline{6}&\underline{9}&\underline{8}&\underline{5}&\underline{9}&\underline{9}&\underline{2}&\underline{3}&\underline{7}\end{array}$$

$$\begin{array}{ccccccccccccccc}1&3&4&3&5&1&2&6&1&3&5&6&9&1&4\\ \underline{8}&\underline{4}&\underline{9}&\underline{8}&\underline{5}&\underline{7}&\underline{7}&\underline{6}&\underline{3}&\underline{3}&\underline{6}&\underline{7}&\underline{9}&\underline{9}&\underline{7}\end{array}$$

2. See if you can do the following subtractions in less than 25 seconds:

$$\begin{array}{ccccc}(a)&(b)&(c)&(d)&(e)\\ 598&6843&73659&74325&51324\\ \underline{483}&\underline{582}&\underline{52174}&\underline{25153}&\underline{47548}\end{array}$$

3. Add the following (see Team Game on next page):

$$\begin{array}{ccccc}(a)&(b)&(c)&(d)&(e)\\ 56&84&96&125&2567\\ \underline{75}&\underline{63}&\underline{78}&\underline{256}&\underline{1326}\end{array}$$

A TEAM GAME

Form two or more teams of the same number each. Let your teacher dictate the parts of the preceding example 3 in order. You add each two numbers; then add that total to the same two numbers above it; then add that total to all numbers above it, and so on until you get the tenth total, which will be the answer. See how many can get each tenth result right. Each right counts 10. Each wrong counts 0. Then total up the scores of the members of each team. Every team should work (*a*) until it is completed or until the teacher says stop; then (*b*), etc.

4. I. R. B. Test [2]. First Trial Scores.

Subtract across. Then add all three columns. Check your results.

564 – 421 =
372 – 241 =
784 – 673 =
248 – 157 =
348 – 256 = ______
— =

Multiply across and then add upward in the last column.

8 × 54 =
10 × 62 =
9 × 37 =
11 × 58 =
12 × 63 = ______

5. I. R. B. Test [3]. First Trial Scores.

Divide across and add the last column. See if you can check.

60 ÷ 6 =
42 ÷ 6 =
72 ÷ 6 =
84 ÷ 6 =
24 ÷ 6 =

Add across from left to right. See if you can find a way to check.

7 + 6 + 9 + 8 + 2 + 4 =
3 + 5 + 7 + 6 + 4 + 9 =
4 + 2 + 6 + 7 + 5 + 3 =
9 + 4 + 7 + 4 + 6 + 7 =
8 + 5 + 3 + 5 + 9 + 8 =

Add and check

352 + 964 + 125 =
796 + 538 + 378 =
573 + 679 + 497 =
895 + 786 + 874 =

Divide

729 ÷ 9 =
128 ÷ 8 =
242 ÷ 11 =
372 ÷ 12 =
420 ÷ 15 =

6. If the average receipts of a bus are $8.75 per trip, what are the receipts for 56 trips?

7. How much change should be given for a $5 bill when each of the following purchases is made?

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
$2.46	$3.15	$1.38	$4.15	$.76	$1.49

8. A clerk sold 1 pound of coffee at 54¢, 5 pounds of sugar for 27¢, and 2 pounds of butter at 56¢. How much change should the purchaser receive from a $10 bill?

9. The sum of two numbers is 5984. One of the numbers is 4325. What is the other number?

10. 28 is how many times 4? How did you work this example?

11. In Mr. Rhodes' store a pound of coffee costs 48 cents and a pound of tea 60 cents. What is the ratio of the cost of a pound of coffee to that of a pound of tea? Make a graph that will show this ratio.

12. The ratio of the cost of a pound of sugar to a pound of coffee is $\frac{1}{8}$, and the ratio of the cost of a pound of cheese to a pound of coffee is $\frac{1}{2}$. What is the price of sugar per pound when coffee costs 48 cents a pound? of cheese? Represent the above relations graphically.

13. The price of a pound of tea will buy 6 pounds of new potatoes, 4 pounds of white beans, or 3 pounds of soda crackers. State the ratio of each of these costs to the cost of a pound of tea. Show these relations by a graph.

14. Draw bar graphs to represent the cost in your home town of one pound of each of the articles named in problem 13.

15. State the ratios between the prices of different things in your home-town stores, for example, a yard of gingham and a yard of silk, or a baseball and a fishing rod.

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER III

In Chapter III you have actually started your New Mathematics. You have found how ratios and graphs come from fractions. Thus you will discover that other parts of New Mathematics *grow out* of arithmetic or your Old Mathematics.

In addition to the two graphs, which you have drawn in your I. R. B. and on which you will make future records of your school marks and of your height and weight, you have also entered First Trial scores for three tests. Take them now a second time in out-of-school time. If you fail to improve the First Trial scores, try again. When you are satisfied with the results enter the new End-of-Chapter scores.

CHAPTER IV

PICTURES OF LINE RELATIONS IN GEOMETRY[1]

Our study of the last chapter brings us to a still more interesting part of the New Mathematics. This is the use of lines in measuring; it is called *geometry*.

A BASEBALL FIELD

This picture shows "Jake" Stahl of the University of Illinois knocking a home run with three men on bases in a game with the University of Michigan. Stahl later became a professional and piloted the Boston Red Sox to a world's championship.

What Geometry Is. — This name *geometry* comes to us from two old Greek words meaning *earth measurement*. Jack Wood and James Marsden used geometry when they measured the ground in laying out their baseball diamond and football field. So did Ruth

---

[1] TEACHER'S NOTE: Chapters IV to VI may be supplemented, if desired, by the additional work in geometry on pages 337–342.

Malcolm and Mabel Johnson when they made a tennis court. In similar ways we all make almost daily use of "earth measurement." Geometry is one of the most useful branches of mathematics.

Lines are the first tools of geometry. Every figure, every diagram starts with a line. You have used lines all your life. So without being aware of it you have always used geometry. Now you are going to learn what some of the other tools of geometry are and how to use them in solving problems in mathematics.

The Straight Line. — You all know what a straight line is even though it is hard to define. In the country when a boy wants to go swimming he usually goes "straight across lots" to the swimming hole instead of going around by the road. This is because *a straight line is the shortest path between two points.* Do you sometimes "cut corners" in the city? What is meant by "jay-walking"? Why do people do it?

A Straight Line

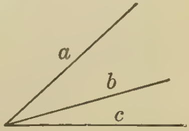
The word "straight" comes from an old word meaning "stretched." A stretched wire is a good illustration of a straight line. If you drive a nail in the corner of your house and another in the corner of your garage, how many straight lines can there be connecting these points? Because there can be only one straight line joining the same two points we often say that *two points determine a straight line.*

How many "sights" are there on a gun barrel? If you sight along the gun barrel until the target is "in line," what does that mean?

A line like the line of sight we have just mentioned is of unlimited length. When we want to emphasize that a line is limited like one joining two points and not extending beyond them, we call it a *line segment.*

How Line Segments Are Named. — The line segment which joins the points A and B as A ———— B is usually called simply "the line AB" or "the line BA."

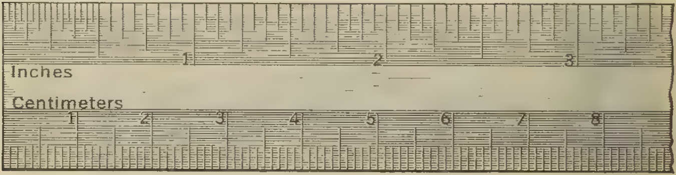
Sometimes a single letter is used to indicate a line segment. In the accompanying diagram we have the lines a, b, and c meeting at a point.



Old and New Means of Measuring. — Make a dot or point on your paper and put the letter A beside it. We shall use this letter to represent the name of the point, and call it "the point A." A short distance away make another point, B. Draw a straight line segment between A and B. Can you do it freehand? If not, use a *ruler.* A ruler is usually a bar of wood or celluloid having a straight edge. For this reason it is often called a *straightedge.* The diagram on the following page shows a part of a foot ruler.

For what other purpose is a ruler used besides drawing straight-line segments?

What part of an inch is the smallest division on the upper scale? What part of an inch is each of the other divisions? How many inches in a foot? How many feet in a yard? How many feet in a rod? How many feet in a mile? If you cannot answer these questions, learn the table of *linear measure* in the back of the book.



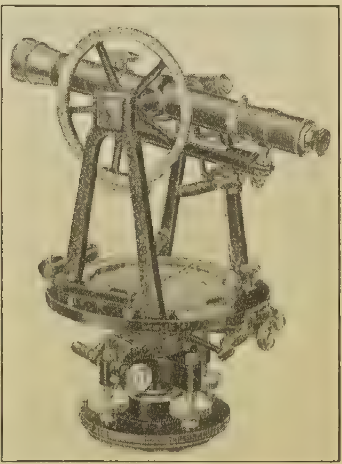
Now look at the lower edge of the ruler. It has different divisions from those of the upper edge. The large divisions are called *centimeters* and the smaller divisions are called *millimeters.* Look at it carefully and tell how many millimeters make a centimeter. This method of measuring distances is called the *metric system.* It is used throughout a large part of the world and is coming to be used more and more in the United States. Learn the table of *metric linear measure* in the back of the book.

MEASURING LENGTH

What metric unit is nearest to a yard? to a mile?

Estimating and Measuring Lengths of Lines. — Using a folded sheet of paper as a straight edge draw a line that looks three inches long. Measure it with your ruler. When you measure a line be sure to place the ruler so that the very end of an inch or centimeter space falls exactly on the end of the line to be measured. If your ruler is old and the end is worn, you will measure more accurately to start, not at the end, but at the point marked 1.

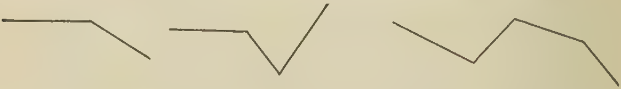
How long is the line that you drew? How much was your error? Try it again. Did you do better this time? First estimate and then measure the length and width of your book by both linear and metric measures. Do the same for the top of your desk. The picture on the opposite page shows Mary Kiernan measuring the length of a crayon box. How long is the crayon box in your room?



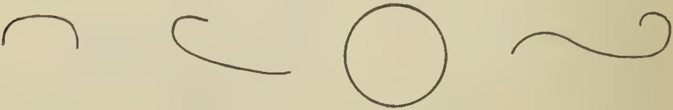
A Transit

Measuring Longer Lines. — When we want to measure a long line like the length of a street or the distance between two towns we get *surveyors* or *civil engineers* to measure it. Surveyors use in place of a ruler a steel tape 100 feet long, divided into feet and *tenths* of a foot. An instrument called a *transit* is used to keep them in a straight line.

Broken and Curved Lines. — In the chapter on graphs we had pictures of broken lines but we did not define them. A line made up of several line segments running in different directions is called a *broken line.*



A *curved line* or *curve* is a line no part of which is straight.



The Circle. — The most important curved line is called a *circle.* It is a *closed curve which is everywhere the same distance from a point inside called the center.* Can you draw a good circle free-hand? Try it. Now try it again, using a circular object such as a coin. Name other circular objects with which you might draw circles. Could you draw a circle of any size that you might want that way?

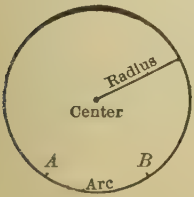
Pen Point

Pencil Point

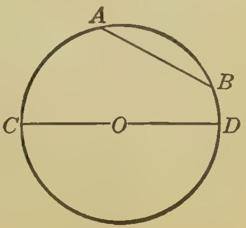
Compasses. — Dick Clark had so much trouble drawing circles of the sizes he wanted that his father gave him a pair of compasses. This consists of two legs of the same length, with a movable joint between them. One leg ends in a sharp mental point, the other in a pencil or pen point.

Using Compasses

The distance from any point on a circle to its center is called the *radius* of the circle. To draw a circle with a radius of one inch we open the compasses so that the distance between the end points is one inch. (Use a ruler to determine this distance.) Then we thrust the sharp point of the compasses into the paper and turn the pencil point completely around, forming a closed curve. This is a circle. The point where the sharp end pierced the paper is the center of the circle.



Any part of the curve of the circle is called an *arc*. Many arches are arcs of circles. All lines made by the compasses are arcs, when they stop short of a complete circle.

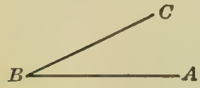


A *chord* is a straight line joining two points on a circle; as the chord *AB*. If a chord passes through the center, it is a *diameter;* as the line CD. A diameter consists of two radii extending in opposite directions from the center.

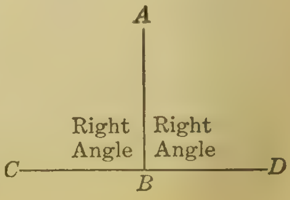
Name some common circular objects.

Angles. — As the hands of a clock rotate, they form various two-sided figures, called *angles.* In the same way, if a radius turns (rotates) about the center of its circle, it makes different angles with other radii of the circle. In the figure, angle ABC shows how much the radius AB must turn about point B to reach the position of CB. The radii AB and CB are called the *sides* of the angle and point B is its *vertex.*

As we cannot conveniently picture an angle as the rotation of a radius, we may represent it by drawing two radii of a circle. The angle is *measured* by the amount of rotation or opening between the two radii. Since all the radii of a circle are drawn from the same point, we may say, then, that *an angle is the figure formed when two straight lines are drawn from the same point.* In naming an angle we usually place a letter at the vertex, and one at the other end of each side, as shown in the diagram. When only one angle is named at a point, we may identify it simply by the vertex letter. Thus this angle may be called "the angle B."



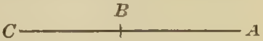
Kinds of Angles. — When one line meets another, as in the diagram, so that the two angles formed are equal, each is called a *right* angle. A right angle is often called "a square

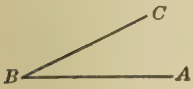


corner.” How many right angles can you see from where you sit?

When two lines form a right angle each line is said to be *perpendicular* to the other. Thus AB is perpendicular to CD.

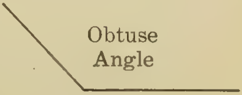
A *straight angle* is an angle whose sides lie in the same straight line but extend in opposite directions. Thus, two radii which form a diameter make a straight angle.

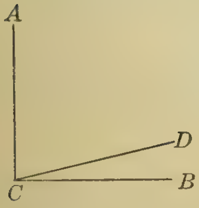




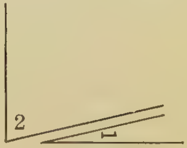
An *acute angle* is an angle smaller than a right angle. *Acute* is another word for *sharp*.

An *obtuse* angle is an angle larger than a right angle and smaller than two right angles. *Obtuse* is another word for *blunt*.

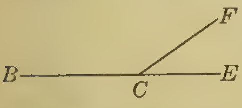




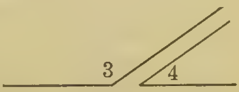
Two lines are *oblique* to each other when they make either an acute or an obtuse angle. All angles except straight angles and right angles are oblique angles.



Two angles are *complementary* if their sum equals a right angle, as angles BCD and ACD or angles 1 and 2.



Two angles are *supplementary* if their sum equals a straight angle, as angles ECF and BCF or angles 3 and 4.



Two complementary angles are called *complements of* each other, and two supplementary angles, *supplements*.

In the accompanying diagram of Rochester streets point out several streets which meet at right angles. Point out at least four acute angles; at least three obtuse angles; half a dozen supplementary angles.

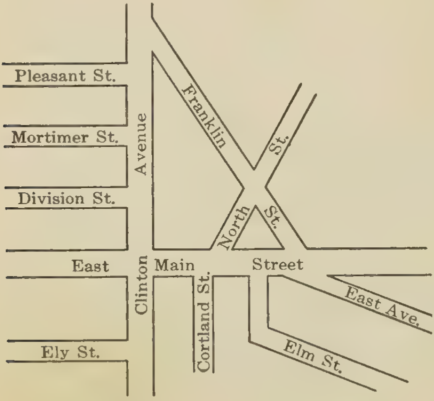


Diagram of Part of Rochester

Geometry Practice [1]

1. Draw a straight line free-hand which you think is 2 inches long. Check your estimate with a ruler.

2. Draw from the same point, six lines, one of which shall be 1 inch long, another $1\frac{1}{2}$ inches long, another $2\frac{3}{4}$ inches long, another 2 centimeters long, another 5 millimeters long, and another 3 centimeters 4 millimeters long.

3. Draw an acute angle; a right angle; an obtuse angle; a straight angle. Letter all of these angles. Try to do a good job of printed lettering.

4. In which of the angles you have just drawn is there a line perpendicular to another line of that angle?

---

[1] Before attempting the following exercises, each pupil should have:

1. A graded ruler.
2. A hard and a medium pencil.
3. A pair of compasses.
4. An eraser.
5. A notebook with unruled paper.

The pupils should do neat, accurate work even if it has to be done over a number of times.

Much of this type of work may well be done in the classroom under the teacher's supervision.

5. Draw a circle which shall have a radius of 1 inch; of $1\frac{1}{2}$ inches; of $\frac{1}{2}$ inch. Which was the hardest to draw?

6. Draw two complementary angles; two supplementary angles.

7. What kind of angle do you find at the corner of one of your books? Name two other objects in the room which have the same kind of angle at the corners.

8. Make a drawing similar to the one on page 84 of a section of your own town or city. Point out and name some of the different kinds of angles that you find in your drawing.

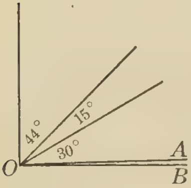
9. I. R. B. Test [1]. Prepare a chart in your I. R. B. for Chapter IV (2 tests). Enter First Trial scores. Have the Steering Committee (teacher ex-officio) give a standard time.

Ruth Malcolm's father wanted to make a table of sales in his business. He made the space for headings $\frac{3}{8}$ of an inch high and allowed $\frac{1}{4}$ inch for each day. Then he made the columns for each department one inch wide and allowed $\frac{3}{8}$ of an inch in depth for the total. Using these measurements, draw a form for Mr. Malcolm, insert the following figures, and find all totals required.

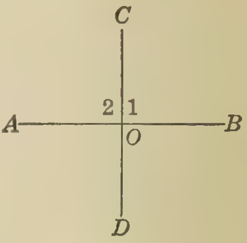
SALES FOR WEEK ENDING OCT. 10, ——

DAYS	DEPT. 1	DEPT. 2	DEPT. 3	DEPT. 4	TOTAL
Monday	$452.28	$563.14	$612.15	$237.85	
Tuesday	342.21	452.26	465.28	210.35	
Wednesday	312.36	483.38	532.38	274.28	
Thursday	412.48	496.76	486.56	296.54	
Friday	426.76	502.58	564.39	212.48	
Saturday	464.84	512.62	614.46	242.24	
Total					

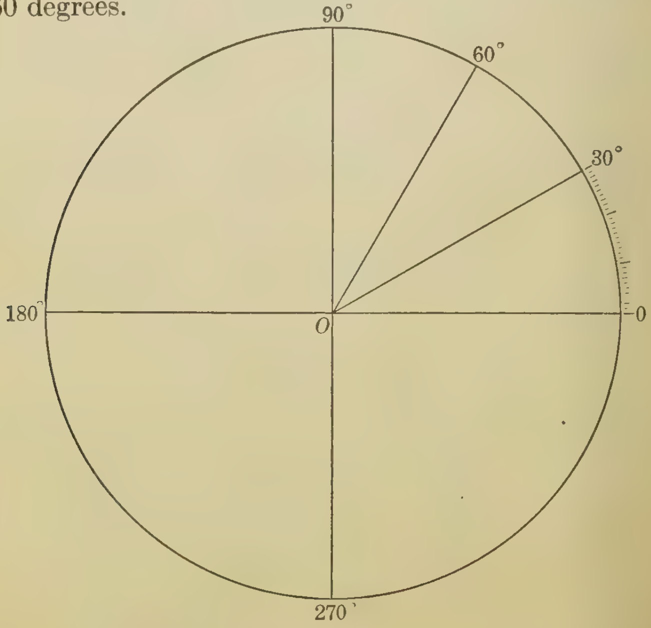
How Angles Are Measured. — Angles are measured by *degrees*. A *degree* is $\frac{1}{90}$ of a right angle.



Here is a right angle divided into degrees. The small division along *OA* is an angle of 1 degree.

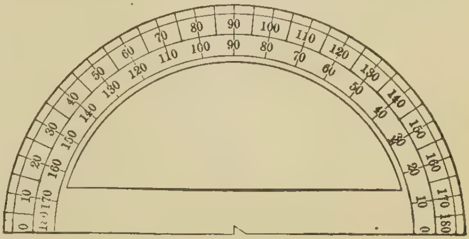


Four right angles placed together like this take up all the space around the point *O*. Hence the sum of all the angles that can be drawn around a point is 360 degrees.



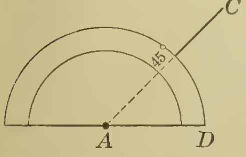
If we draw a circle with *O* for center, the circle measures the sum of all the angles that can be drawn around the point O. So we say that the circle has 360 degrees. The sign that stands for the word *degree* is °, thus 360°. One degree equals 60 minutes (60′) and one minute equals 60 seconds (60″). Learn the table of *angular measure* in the back of the book.

The Protractor is an instrument used to measure angles. You will find one in a pocket attached to the back cover of this book. Usually protractors are made of brass, celluloid, or heavy cardboard, and are in the shape of half a circle (semicircle).

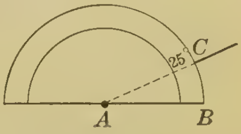


A PROTRACTOR

To measure the angle CAD we place the protractor so that the little point in the middle of its straight edge is at the vertex A, and this edge falls along the side AD of the angle. Where the other side of the angle or an extension of it cuts the circular edge of the protractor we read 45°. Hence the angle CAD is an angle of 45°.

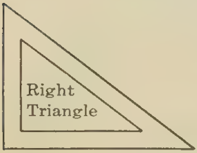


We may use the protractor to construct an angle of any size we wish. For example, suppose we wish to construct an angle of 25°. We first draw a line AB. Then we place the protractor with its central point or notch at A, and its straight edge along AB, as shown in the diagram.

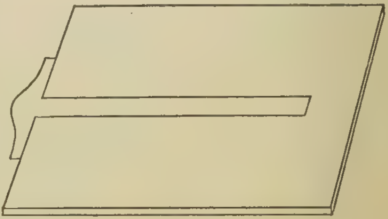


Next make a dot at C where the mark on the protractor indicates 25°. Now we remove the protractor and draw a line from A to C. Then the angle CAB is an angle of 25°.

Draw what looks to you like a right angle. Measure it. How nearly correct were you? Try again by estimating angles of 45°, of 30°, of 60°, of 10°. Have another pupil draw several angles with the protractor, then you try to estimate the number of degrees in each of them. Then you may draw the angles and let him estimate the size of each. A game like this will help you to get a good idea of angles.



Drawing Right Angles. — Have you ever seen a carpenter at work? He uses an instrument called a *square* to draw a right angle. Sometimes a celluloid model of a *right-angled triangle* is used.



Ask one of your friends, who studies mechanical drawing, to show you how he draws a right angle by the help of his drawing board and *T-square*. Do you see why it is called a T-square?

Constructing Angles

1. Draw each of the following angles with your protractor.

20°, 40°, 120°, 60°, 12°, 70,° 270°

2. Cut out and arrange according to size the angles that you have drawn in example 1.

3. Name these angles as to kind, that is, acute, obtuse, etc.

4. What is the complement of each of the above angles which is less than 90°?

5. What kind of angle is formed by the hands of a clock at 3 o'clock? How many degrees has this angle?

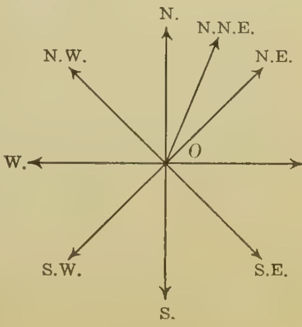
6. Find the size of the angle between the hands of a clock at 6 o'clock; at 9 o'clock; at 5 o'clock; at 2 o'clock; at 12.25. (Use your protractor where necessary.)

7. If a wheel has six spokes equally spaced, how many degrees are there between any two spokes that are next to each other? any two spokes that are separated by one spoke?

Angles and Direction. — When the boys laid out their baseball diamond, they ran the line from the home plate due north to third base. In making the line from home to first at a right angle with the line to third, James Marsden said it ran due east. Was he right? In what direction would the pitcher pitch? In what direction would the shortstop throw from second to third?

The directions, north, east, south and west give us a fine chance to study angles.

In the diagram, what angle is formed at *O* by the north and east lines? The direction called northeast is exactly halfway between north and east as indicated. How many degrees in the angle between northeast and east? between northeast and north?



between northeast and northwest? between northwest and south?

Halfway between north and northeast is a point called north-northeast. How many degrees is that from north? Locate east-northeast, west-northwest, south-southwest, east-southeast.

The Points of the Compass. — In making circles we used a compass that had two points. However, these are not what we mean when we speak of the *points of the compass.* The expression *points of the compass* refers to the directions, north, east, south, and

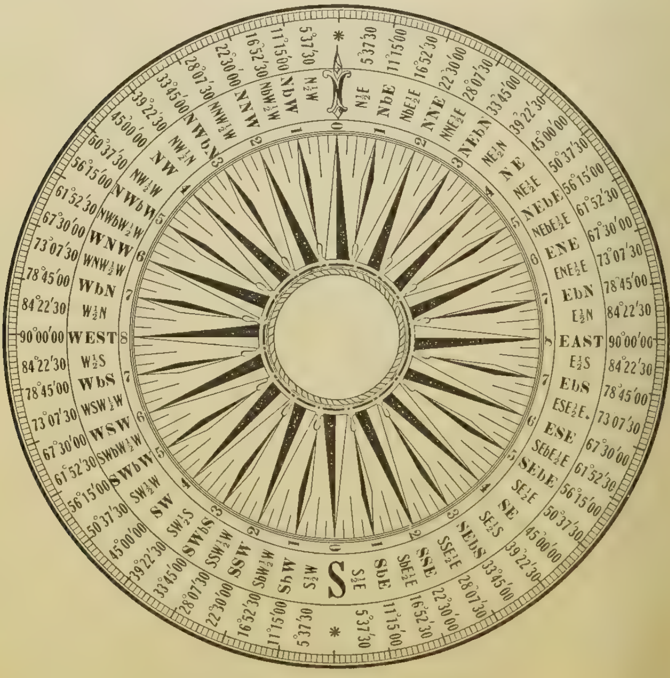
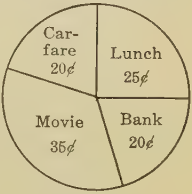


Diagram of a Mariner's Compass

west, which we have just been studying. They are best shown on an instrument called a *mariner's* (sailor's) *compass*, which is not at all like the compasses we have used. All ships have mariner's compasses by which they steer their courses. Opposite is a picture of one.

If you know an old sailor, get him to tell you some *readings* of the compass. If you don't know a sailor, see if you can figure out for yourself the meaning of "north-northeast by north and a half north."

The Circle Graph. — The mariner's compass shows a complete circle divided into 360 degrees. Because of these uniform divisions circles are often used as graphs. Being round like coins, they are often used to show proportionate expenditures. Thus if your father gives you a dollar and you spend 20 cents for car fare, 25 for lunch, 35 for a movie, and put the rest in your bank, you would show it graphically like this:



One Dollar

Let the circle represent the dollar. As 25 cents is one quarter of the dollar, we represent it by $\frac{1}{4}$ of the circle, or 90°. With a protractor we lay off the 90° angle with its vertex at the center. 20 cents is $\frac{1}{5}$ of the dollar, so we use $\frac{1}{5}$ of 360°, or 72°, for the angle opening that shall represent car fare. 35 cents equals $\frac{35}{100}$, or $\frac{7}{20}$, of the dollar and $\frac{7}{20}$ of 360° is 126°, therefore we construct with our protractor an angle of 126°. The rest of the space represents the amount put in the bank.

Making Circle Graphs

Construct circle graphs to represent the following.

1. A boy earns 20 cents mowing a lawn, 25 cents delivering packages, 10 cents selling papers, 30 cents running errands, and the other 15 cents of a dollar by carrying a message for a neighbor.

2. The boy in example 1 spent his dollar as follows: 10 cents for an ice cream soda, 25 cents for a ball game, 30 cents for a movie, 15 cents for candy, and put the rest in his bank.

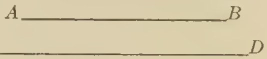
3. In Memorial High School there are 600 students. Of these 240 are freshmen, 160 are sophomores, 120 are juniors, and the rest are seniors. How many seniors are there? Draw a circle graph to show how this school is divided into classes. What is the ratio of the number of sophomores to the number of freshmen? of the number of seniors to the number of freshmen?

4. Jack's brother in High School says he studies three hours at home every day on his five subjects. He spends one hour on mathematics, $\frac{3}{4}$ hour on science, $\frac{1}{2}$ hour on English, $\frac{1}{2}$ hour on music, and history one-fourth of an hour. Draw a circle graph to show how he divides his study time. Which subject to you think is hardest for him? Which one do you think the easiest? *Hint:* Let an angle of 120° represent an hour.

5. It cost $1200 a year to send Harold Wagner to college. Of this $400 was for tuition, $300 for board, $200 for room, $100 for books and laboratory fees, $50 for traveling expenses, and $150 for incidentals: Draw a circle graph to show how his various college expenses were divided.

6. A dry-goods merchant sold in one year goods amounting to $50,000. He paid $30,000 for the goods, $5000 for rent, $8000 for clerks, and $2000 for light, heat, etc. The rest of the $50,000 was profit. Draw a circle graph representing the $50,000 receipts, and show by the graph how it was divided.

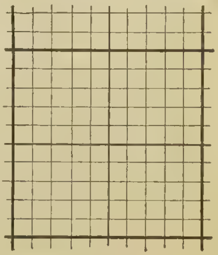
Parallel Lines. — In our circle graphs and study of the points of the compass we learned more about angles. But some lines do not form angles. Two straight lines on the same plane (flat surface) are called *parallel*, if they would never meet however far extended.



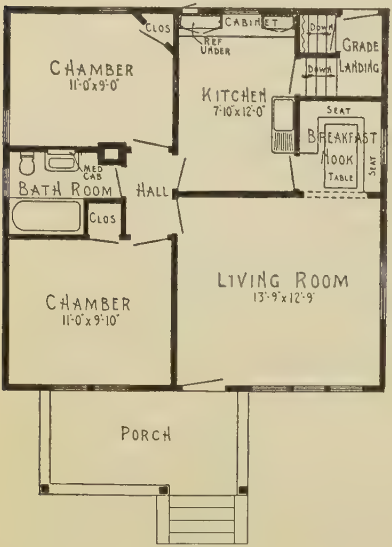
Parallel lines are the same distance apart at all points. The rails of a straight railroad track, the opposite edges of your book, and the edges of the window casing are examples. Look around the room and see how many parallel lines you can find.

Like right angles, parallel lines are one of the commonest figures of geometry. We are now going to study something in which both right angles and parallel lines are constantly used.

Squared paper which we used to make graphs on page 54 is an illustration of parallel lines and right angles. It is a great help in drawing. In the pocket at the back of the book you found a sample of this kind of paper. Show how you can use it like a ruler for measuring. Draw on the squared paper a right angle ABC. Now draw from B a line BD that will divide the right angle into two equal angles. How can the point D be located? Draw a line EF parallel to BC. How did you locate the points E and F?



Drawing to Scale. — Before a building is built the architect draws plans for it. He cannot conveniently make a drawing as large as the floor. If the floor is to be 30 feet wide and 40 feet long and he makes a 4-inch line to represent the length, how long will he make the line which represents the width? Will he make the angles in the drawing right angles as they are in the floor?



A Floor Plan Drawn to Scale

A drawing made like this is said to be *drawn to scale*. That means that all the angles are just like those in the building to be constructed, but a smaller unit is used for the lines. The drawing is a graph in which the ratio of the lines is the same as in the proposed building.

Are the maps in a geography drawn to scale? Open your geography at the map of your state and see if it tells you what scale is used. Suppose that it says that the scale is 100 miles to the inch, what does that mean? On that map a distance of $\frac{1}{10}$ inch would represent how many miles?

Drawing to Scale on Squared Paper

1. Jack Wood drew on squared paper a plan of his schoolroom floor, letting one inch represent 5 feet. Try it for your room. Locate the door, the windows, and the teacher's desk.

2. If possible, obtain a map of your village or city. Draw to scale a map of your immediate section on a scale four times as large as this map.

3. Copy the map on page 84, using a scale of two to one.

4. Construct on your squared paper four right angles, using a scale of $\frac{1}{2}$ inch to 1 foot.

(*a*) If one side is 3 feet and another side is 4 feet.
(*b*) If one side is 6 feet and another side is 8 feet.
(*c*) If one side is 12 feet and the other side is 16 feet.
(*d*) If one side is $1\frac{1}{2}$ feet and the other side is 2 feet.

5. Mr. Henderson took Ethel on an auto trip through Missouri, Oklahoma, and Texas. He used an automobile map printed to a scale of $28\frac{1}{2}$ miles to the inch. How many miles would each of the following distances represent: 4 inches? 6 inches? $2\frac{1}{2}$ inches? 8 inches? $6\frac{1}{2}$ inches? 10 inches? 15 inches?

6. About how many miles would $\frac{1}{4}$ inch represent?

7. If you started out on an automobile trip and used the map in problem 5 as your guide, how many inches of the map would you measure to make a first day's trip of 180 miles? If you averaged 22 miles an hour, how many hours would you require for the trip?

8. On a certain map of Rhode Island the map-maker used a scale of 10 miles to the inch. If the greatest length of the state is 48 miles and the greatest width is 37 miles, how many large squares and how many small squares would you use on your squared paper to represent each of the above distances? How many miles would 2 large squares and 2 small squares together represent?

9. With the same scale as given in example 8, how many miles would $6\frac{1}{2}$ inches represent? $8\frac{1}{4}$ inches? $2\frac{1}{4}$ inches? $4\frac{1}{2}$ inches?

10. Draw on your squared paper two parallel lines $\frac{1}{5}$ of an inch apart to represent 800 miles and 400 miles, if 1 inch is to represent 200 miles. If you were to draw these lines at a scale $\frac{1}{4}$ inch for 200 miles, how long would each line be? Draw them.

11. Draw on your squared paper two parallel lines to represent 6 million and 4 million people if 1 inch is to represent 2 million people.

12. Draw on the squared paper two parallel lines to represent the weekly wages of two girls receiving $15 and $20 respectively, if 1 inch is to represent $5.

REVIEW TRIAL OF I. R. B. TESTS, CHAPTER III

Take the third trial of the three tests of Chapter III. On the chart where you entered the First Trial and End-of-Chapter scores, record your new scores in the Review Trial Column. Keep trying the tests until you make new scores worthy of yourself and of your Improvement Record. Read once more the directions given in the Review Trial of Chapter II.

If you wish to add a new feature to your I. R. B., enter some of the *italicized* words of this fourth chapter and give your own definitions of the terms. Turn back through the pages of this chapter to find the terms; for example: geometry, straight line, line segment, broken line, curved line, circle, radius, chord, diameter, angle, vertex, right angle, perpendicular, straight angle, acute angle, obtuse angle, complementary and supplementary angles, parallel lines, etc. You will find other italicized terms which you may want to add to your I. R. B. record.

General Review

1. In the line *ACB* what are the parts *AC* and *BC* called?

2. How do you prove your work in subtraction? In addition? In multiplication? In division?

3. If you were told the weight of six cows, how could you find their average weight?

4. If you were given the total sales of 5 clerks in a store for one week, how could you find their average sales for the week?

5. If the divisor is $6\frac{1}{2}$, the quotient is 8, and the remainder is $2\frac{1}{2}$, how would you find the dividend? What is it?

6. Mr. Brown earned on an average $4.75 a day for a month of 26 working days. How could you find his monthly earnings? Find them.

7. Mr. Warner earned $2580 in one year. His expenses were $1975. How could you find his savings? Find them.

8. If you were told that $\frac{1}{4}$ of the distance from San Francisco to Seattle was $179\frac{1}{2}$ miles, how could you find the total distance between them? Find it.

9. How many degrees are there in $\frac{1}{6}$ of a right angle? in $\frac{1}{9}$ of a right angle? in $\frac{1}{3}$ of a right angle? in $\frac{1}{4}$ of a right angle? With your protractor draw each of these angles.

10. If 15° equals $\frac{1}{12}$ of a certain angle, how many degrees in that angle? What kind of angle is it?

11. If two straight lines meet at a point so as to form an angle of 90°, what is the relation of each line to the other?

12. If 1 decimeter is approximately 4 inches long, estimate in decimeters the length and width of a desk 60 inches by 32 inches.

13. If 1 centimeter is approximately $\frac{2}{5}$ of an inch long, estimate in centimeters the height of a boy 4 feet 6 inches tall.

14. One kilometer is approximately $\frac{5}{8}$ of a mile. How many miles is it from New York to Chicago, if it is approximately 1600 kilometers?

$3\frac{3}{4} : 2$

15. The marathon run is 22 miles, 160 rods. How many kilometers (approximately) is that?

16. Add: $\frac{1}{5}$ of 250, $\frac{1}{8}$ of 240, $\frac{1}{7}$ of 637, and $\frac{4}{9}$ of 360.

17. How many times 2 are $13\frac{1}{3}$?

18. If John can walk 13 miles while Henry is traveling $32\frac{1}{2}$ miles, how far can Henry travel while John is walking 1 mile?

19. From a piece of cloth containing 25 yards, a tailor cut 8 suits of clothes containing $2\frac{3}{4}$ yards each. How many yards remained?

20. If the distance from home plate to second base is $1\frac{2}{5}$ (approximately) times the distance from home to first base, which is 90 feet, how far is it from home to second base?

21. George's height is $5\frac{2}{3}$ feet. His father's height is $6\frac{1}{12}$ feet. By how much does the father's height exceed George's height?

22. A real estate agent sold a house for Mr. Evans and obtained $10,000 for it. If the real estate agent received $500 for his work in selling the house, what part of the selling price did he receive?

23. A man owning a building worth $102,000 insured it for $68,000. For what part of the value of the building was it insured?

24. If 1 kilogram equals $2\frac{1}{5}$ pounds, how many kilograms will a cubic foot of water weighing $62\frac{1}{2}$ pounds weigh?

25. Which is heavier and how much, 10 kilograms or 336 ounces avoirdupois?

26. Ruth Malcolm could not think of the meaning of *acute* and *obtuse*. Can you?

27. The Eiffel Tower in Paris is about 300 meters high. The Woolworth building in New York City is 792 feet high.

Which is the higher and how many feet higher? See tables in appendix.

THE EIFFEL TOWER IN PARIS

28. Mr. Davis had \$564.46 in the bank in the morning. He checked out during the day \$56.45, \$123.75, \$21.72, \$87.40, \$96.37, and \$7.15. How much had he left in the bank that night, if he did not deposit any money during that day?

29. If an airplane flies 182 miles in $3\frac{1}{4}$ hours, how many miles would it fly at the same rate in 6 hours?

30. If an automobile averages 18 miles to 1 gallon of gasoline and the gasoline costs 20 cents a gallon, how much will the gasoline cost for a trip of 600 miles?

31. The minimum (or lowest) temperatures for the 31 days of a certain July were 64, 61, 61, 61, 61, 67, 71, 70, 69, 69, 66, 68, 73, 68, 64, 67, 65, 58, 58, 61, 65, 73, 70, 68, 71, 63, 62, 58, 60, 68, and 72 degrees. Find the average (or mean) minimum temperature.

32. I. R. B. Test [2]. First Trial scores. Have the Steering Committee set a standard time. Copy the following and find all required totals.

DELIVERIES PER WEEK

DAY	TRUCK #1	TRUCK #2	TRUCK #3	TRUCK #4	TRUCK #5	TOTAL
Mon.	234	317	256	161	217	
Tues.	231	313	378	174	243	
Wed.	267	284	275	123	197	
Thur.	294	278	341	147	207	
Fri.	314	368	397	224	274	
Sat.	212	236	254	125	198	
Total						

What do you note especially about truck number 4?

What type of articles do you suppose this truck might be delivering?

Can you think of any value that the above record might have for the manager of all of these trucks?

Make a bar graph showing the total deliveries of each truck.

33. If you knew the cost of a bolt of cloth and the price per yard, how could you find the number of yards?

34. What would you have to know to find the average weight of several truck loads of stone? How would you do it?

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER IV

In this chapter you have made a start on a new branch of mathematics called "Geometry." In a former chapter you learned how graphs are line pictures, lines of ratios, or fractions. Now you know that geometry also is a study of pictures of all kinds of lines and other figures which lines make.

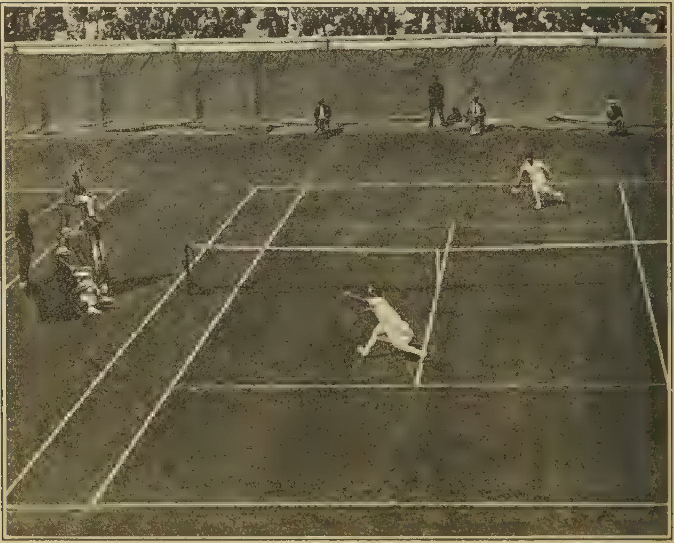
You have recorded some geometric terms in your I. R. B. and taken two short tests of *everyday* addition. Try these tests again in out-of-school time. Enter your second scores (be sure that they are equal to or better than the First Trial scores) in the End-of-Chapter column.

CHAPTER V

USING GEOMETRY AS A TOOL

In the preceding chapter we saw how common geometric forms are. We have always known lines, angles, and circles. Now we are going to *use* them as tools in building up our knowledge of the New Mathematics. While we shall learn a new term or two, most of our work will be using what we have already learned.

What Is Meant by Bisect. — Bisect means to *cut*



A TENNIS COURT

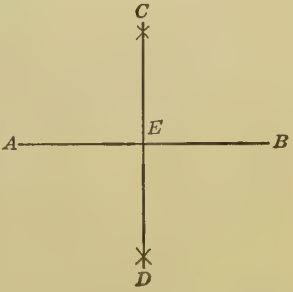
This shows two of America's leading women players, Helen Wills in the foreground and Molla Mallory in the farther court.

in two in the middle. When Ruth and Ethel planned their tennis court they made it 78 feet long. To find where to put the net, they had to *bisect* one of the 78-foot lines. This they did by measuring 39 feet from the end of the line. Thus the line was bisected or *cut in half.*

With the aid of your ruler and protractor draw a diagram of a tennis court to any convenient scale. The length is 78 feet. The width of the double court is 36 feet, the singles 27. The service line is 21 feet from the net. You must bisect the length to place the net, and the service line must be bisected to get the right and left courts.

Using Geometry to Bisect. — Of course we can bisect a line by arithmetic, measuring, dividing, and measuring again. Outdoors with a tape this is all right, but for our diagrams and school work and many other practical uses geometry has a shorter way.

Suppose that we wish to bisect a line segment such as AB. We do not measure the line, but instead, we use the compasses. We open them so that the distance between the points is *greater* than $\frac{1}{2}$ the length of AB. Then placing the sharp point of the compasses at one end A, we draw arcs above and below the line. Next leaving the compasses unchanged, we place the sharp point at the other end B and draw arcs above and below the line, intersecting (cutting) the first arcs at C and D. With the ruler draw the line CD crossing AB at E. You have bisected AB at E. Prove this by measuring. Why must the opening of the compasses be greater than $\frac{1}{2}$ AB?

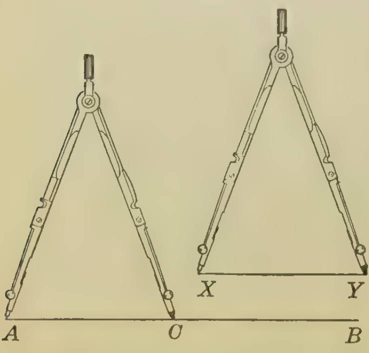


Using Compasses to Bisect Lines

Draw the lines indicated below and bisect them with the aid of your compasses. Then using your ruler, measure the whole lines and check to see if the halves into which the compasses bisected the lines are actually equal.

1. The length and width of this book.
2. A 7-inch line and a 3-inch line.
3. Lines equal to your longest and shortest pencils.
4. The width and half the length of a sheet of paper.
5. A 9-inch line and a 5-inch line.
6. Any line that you may draw.
7. A line one half as long as the one in example 6.

Another Use of the Compasses. — In our tennis court diagram we used the ruler to make the service line on a scale representing 21 feet from the net. Suppose we wanted to place the other service line without using a ruler. To do this, place the point of the compasses at the bisection (middle) of the net line, letting the pencil point fall at the bisection of the service line. Then swing the compasses around to the opposite side and draw an arc across the line bisecting the service court. This measures our 21 feet for the other service line.



If you wish to mark off on a longer line *AB* a segment equal to a shorter line *XY*, first place the sharp point of the compasses on X and open the compasses until the pencil point rests exactly on Y. Then without changing the opening of the compasses, place the sharp point on A and swing the compasses so as to make a little arc across AB at C. Then $AC = XY$.

Measuring by Compasses

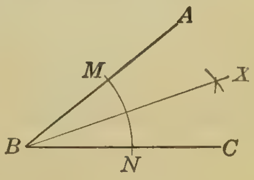
1. On a large sheet of paper draw a line the length of this book. Then with the compasses mark on this line another line segment equal to the width of the book. What is the approximate ratio of the two lines?

2. Draw a line 7 inches long. At one end mark off with the compasses a segment of 3 inches; at the other end a segment of 1 inch. What is the easiest way to draw another line segment equal to the balance of the 7 inches when 3 inches and 1 inch have been cut off at either end?

3. Draw two line segments equal to the longest and shortest pencils which you have. Then on the longer line represent the difference between the two by marking off the length of the shorter.

How to Bisect an Angle. — Suppose that we wish to bisect the angle ABC. Here again we may use the compasses instead of the protractor.

Place the sharp point of the compasses on B, the vertex of the angle, and draw an arc cutting BA at M and BC at N. Then with an opening of the compasses greater than one half the distance from M to N place the sharp point on M and draw an arc. With the same opening, place the point on N and draw another arc crossing the first arc at X. With the ruler draw BX. Then angle ABX = angle XBC, and hence the angle

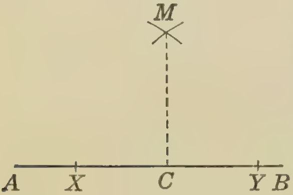


ABC is bisected by the line *BX*. Prove this with your protractor. Notice that we do not need to know how many degrees there are in the angle *ABC* in order to bisect it.

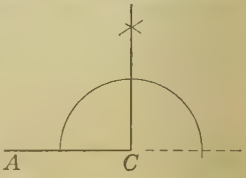
Bisecting Angles by Compasses

Construct a number of different-sized angles. Bisect them with the compasses; then check up your work by actually measuring them with your protractor. You might use 90°, 45°, 30°, and 180° angles to start with.

How to Construct a Right Angle. — All the corners of our tennis court must be square. All the angles must be right angles. Compasses may be used instead of a protractor when we want to make a right angle. We have a line segment *AB*, on which we wish to make a right angle at the point *C*. This is the same as erecting a perpendicular at the point *C*, because perpendiculars make right angles.



Construct a right angle with compasses and ruler as follows. Place the sharp point of the compasses at *C*, and swing an arc crossing the line *AB* at two points *X* and *Y*, Then open the compasses a distance greater than half of *XY*, and placing the sharp point on *X* draw an arc. Keeping the same opening, place the sharp point on *Y* and draw another arc, crossing the first one at *M*. With the ruler draw *MC*. Then *MC* is perpendicular to *AB*, and angles *MCA* and *MCB* are right angles. Check them with your protractor. If the point *C* is at the end of the line, we must first extend the line as shown in the figure.



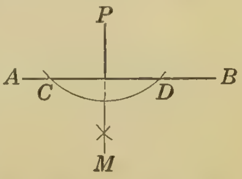
Shorthand Symbols for Angles and Arcs. — One purpose of the New Mathematics is to make our work easier and simpler. So for convenience we use a symbol to stand for the word *angle*. It is a little angle like this, $\angle ABC$. We read this "the angle ABC." Likewise $\overset{\frown}{AB}$ is read "the arc AB."

Reading Shorthand

Read each of the following:

1. $\angle BCA$. **3.** $\angle a$. **5.** $\angle x$. **7.** $\overset{\frown}{BD}$. **9.** $\angle EFD$.
2. $\angle COA$. **4.** $\angle 1$. **6.** $\overset{\frown}{AC}$. **8.** $\angle c$. **10.** $\overset{\frown}{HK}$.

Dropping a Perpendicular from a Point Outside a Line. — Mr. Marsden wanted to make a walk from his front door at right angles to the street. Knowing that James was studying geometry Mr. Marsden asked him how he thought they could be sure about the angles. James drew a diagram showing him how to do it.



He let the line AB represent the street and the point P the middle of the front door. Then he placed the sharp point of the compasses on P and using an opening long enough, swung an arc cutting AB at *two* points, C and D. Next placing the sharp point of the compasses, first on C, then on D, he drew arcs intersecting at M, using the same opening for each. With the ruler he drew PM. PM was the perpendicular to AB, as he proved by both the protractor and the T-square.

In geometry we can drop a perpendicular *up!* Make the point below the line and try it.

Using Geometry as a Tool

1. Draw a line AX. On it mark a point O. At O construct a perpendicular line OC. The $\angle AOC$ should be a right angle.

2. Measure the $\angle AOC$ in example 1 with the protractor. How many degrees in it? Is your work accurate?

3. Bisect the $\angle AOC$. Call the bisecting line OM. How many degrees in $\angle AOM$? In $\angle MOC$?

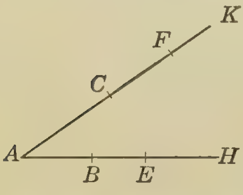
4. How can you construct an angle of $22\frac{1}{2}°$? Try it.

5. Draw a line segment 3 inches long. Bisect it. Bisect each half again. How many equal segments have you now? How long should each be? Measure them with the ruler. How accurate is your work?

6. How could you divide a line segment into eight equal parts?

7. How could you divide an angle into four equal angles? into eight equal angles?

8. How do the angles BAC, EAF, and HAK compare in size? Why?



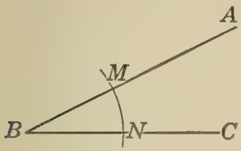
On what does the size of an angle depend?

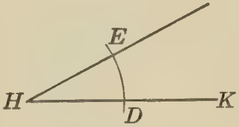
On what does it *not* depend?

9. Draw the line AB. Erect a perpendicular at its middle point and one at each end. Do these lines seem parallel?

10. At the middle point C of the line AB draw a line CD which you *think* is perpendicular to it. From D drop a perpendicular to AB. Was your estimate correct?

Making an Angle Equal to a Given Angle. — In the study of triangles, which we take up next, we not only measure lines, bisect angles, and erect perpendiculars, but we must learn to construct an angle equal to any given angle. This is how we do it.





Let *ABC* be the given angle. We take the line *HK* for one side of our new angle, and decide to use *H* for its vertex.

Place the sharp point of the compasses on *B* and draw an arc cutting the sides of the angle *ABC* at *M* and *N*. With the same opening, place the point on *H* and draw a similar arc cutting *HK* at *D*. Set the compasses at *N* and open them so that the pencil point is at *M*. Using this opening, place the point at *D* and draw an arc cutting the other arc at *E*. Draw *EH*. Then $\angle EHK$ is equal to the $\angle ABC$. Prove this with your protractor.

Constructing and Testing Angles

1. With your protractor draw an angle of 35°. Now draw a straight line and without the protractor but with the compasses construct an angle of the same size at some point on the line.

2. Draw an obtuse angle. Construct with the compasses an angle equal to it.

3. Test the accuracy of your work by measuring the angles with your protractor.

4. Draw a line segment, and mark down a point outside of it. Construct a perpendicular line from that point to that line. With your protractor measure the two angles formed. Are they right angles? Are they equal?

Fixing the Habit of Systematic Review

Try to form the habit of systematically reviewing back work which you know is a stepping stone to new work. "Watch your step," both the last step and the next step. Remember that system will save time and trouble *for you*. "Do it now" is the only sure way to form the good habit of systematic review.

1. With your protractor construct the following angles:

30°, 120°, 45°, 150°, 50°.

2. What is the complement of 30°? of 45°? of 50°? of $66\frac{2}{3}$°? of 89°?

3. What is the supplement of 120°? of 150°?

4. Bisect each of the angles in example 1 without a protractor.

5. I. R. B. [1] Construct a broken-line graph of the following yearly incomes. Choose a convenient unit on the horizontal line for the time and on the vertical line for the money. Copy your graph in your I. R. B.

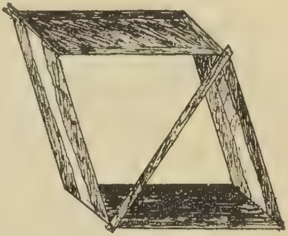
YEARLY INCOME

Age	14	16	18	20	22	24	25
High School Training	In School	In School	$500	$750	$1000	$1150	$1550
No High School Training	$200	$250	$350	$470	$575	$600	$688

Does a high school education pay?

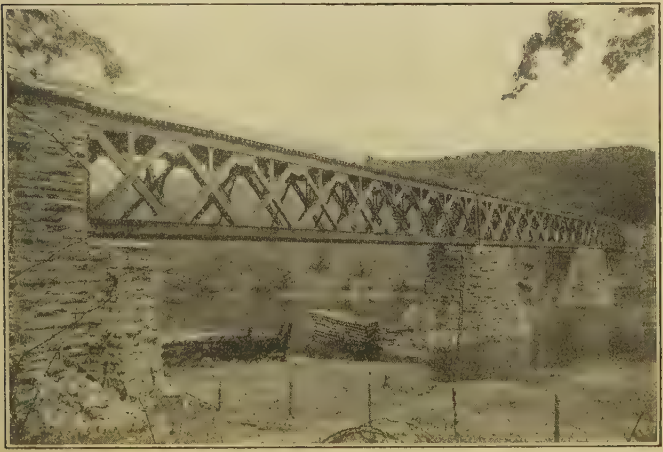
If you want to get ahead and stay ahead, you must use your head and stay on the school job.

Triangles. — On the school picnic the girls made a table by setting a rickety box on its side. The box had no top or bottom, so it settled and finally broke down. George Burns set it up again and nailed a stick slantwise across the open top so as to make a triangle. This time it held up. Can you tell why?



Making the Box Rigid

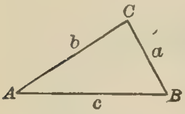
The beauty of a triangle is that it is stiff and rigid. Steel bridges have to be rigid; they always show many triangles. It is this same rigid or fixed quality which makes triangles so valuable in mathematics, especially for fixing points. In much of the work we have just been doing we have really been using triangles, even if we did not always draw the sides.



A Rigid Steel Bridge

See how many triangles you can count in the first span.

A *triangle* is a plane[1] figure bounded by three straight lines, like the figure ABC.



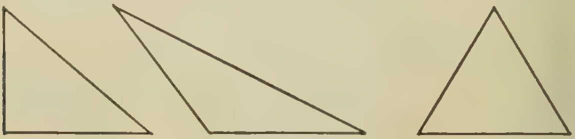
The three lines are called the *sides* and the points where they meet are called *vertices* (each one is a vertex). Thus a triangle has three sides, three vertices, and three angles.

Kinds of Triangles. — When the sides and angles differ in size, they make different kinds of triangles.

An *acute triangle* is a triangle in which all three angles are acute.

An *obtuse triangle* is a triangle in which one angle is obtuse.

A *right triangle* is a triangle in which one angle is a right angle.



RIGHT △ OBTUSE △ ACUTE △

The *hypotenuse* is the longest side of a right triangle. Which angle is it opposite? How many acute angles are there in a right triangle? in an obtuse triangle?

An *equilateral triangle* is a triangle which has all of its sides equal. Equilateral triangles are also *equiangular*, that is, all three angles are equal.

An *isosceles triangle* is a triangle which has two sides equal.

A *scalene triangle* is a triangle whose sides are all unequal.

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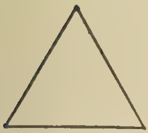
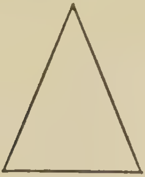
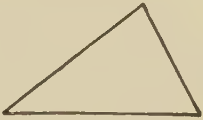
[1] By a *plane* we mean something smooth and flat like the top of a desk or smooth pond. Can you mention some other examples of planes?



Equilateral △
also
Equiangular △



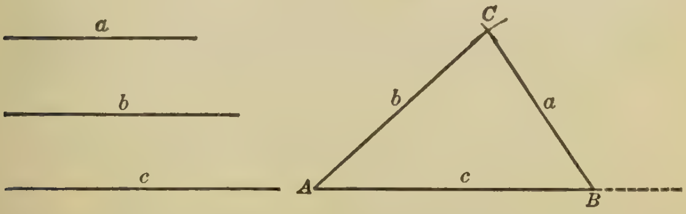
Isosceles △



Scalene △

How to Construct Triangles. — We have learned how to draw lines of a definite length and how to make angles of a definite size. Now we are going to put these things together to make different kinds of triangles.

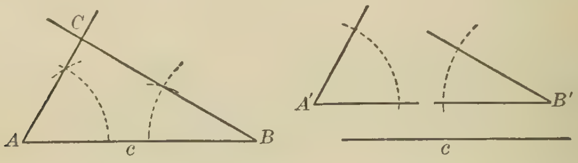
(*a*) *Constructing a Triangle with Three Given Sides.* — If we wish to make a triangle whose sides are equal respectively to three line segments *a*, *b*, and *c*, we first draw a line *AX* of any convenient length. From *A* we mark off with the compasses a distance *AB* equal to the longest segment *c*. Then we open the compasses a distance equal to the segment *b* and placing the sharp point at *A* we swing an arc as shown in the diagram. Finally with *B* as center and a radius equal to the segment *a* we draw an arc crossing the first arc at the point *C*. Then we draw the straight lines *AC* and *BC*. The triangle *ABC* has sides equal to the given lines.



With your ruler measure the line segments *a*, *b*, and *c*, and then measure the sides of the triangle and check the work.

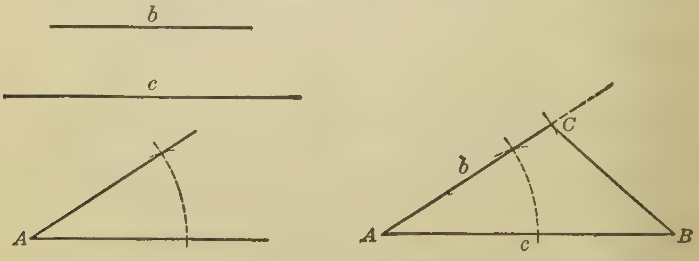
Can you construct a triangle if $a + b$ is smaller than c? Why? Would it be possible if $a + b$ just equaled c?

(*b*) *Constructing a Triangle when One Side and Two Adjoining Angles Are Given.* — Suppose you wish to construct a triangle with given a side c and two given angles A' and B'.



On any convenient line mark off a segment AB equal to c. Then at A construct an angle equal to the given angle A', as you did on page 109, and at B construct an angle equal to the given angle B'. Extend the sides of these angles until they meet at C.

Then the triangle ABC contains the given parts. Using your protractor and compasses, check your construction.



(*c*) *Constructing a Triangle When Two Sides and the Angle between Them Are Given.* — Suppose we wish to construct a triangle when we have given the angle A and its two sides b and c.

On any line mark off from A a line segment equal to c.

At A construct an angle equal to the given angle A. On the other side of this angle mark off a line segment equal to *b*. Draw *CB*. Then the triangle *ABC* contains the given parts. Check this with ruler and protractor.

Constructing Triangles and Measuring the Angles

1. Construct a triangle whose sides are 1 inch, $1\frac{1}{2}$ inches, and 2 inches. What kind of a triangle is this?

2. Construct a triangle in which each side is 1 inch. What is the name of this kind of a triangle?

3. Measure the three angles of your figure in example 2 by the aid of your protractor. What do you observe? As a triangle whose angles are all equal is called an *equiangular* △, an —— triangle is also equiangular.

4. Construct a triangle in which one side is one inch and each of the other sides is $1\frac{1}{2}$ inches. What kind of a triangle is this?

5. Measure each of the angles of your figure in example 4 with your protractor. What do you observe about two of the angles? Which two?

6. Construct a triangle whose sides are 3 cm., 4 cm., and 5 cm. Measure the largest angle. What kind of an angle is it?

7. Give two names by which the triangle you have just constructed in example 6 is called.

8. Construct a triangle whose sides are 6 cm., 8 cm., and 10 cm. What kind of a triangle does it appear to be? Measure the largest angle.

9. Construct a triangle whose sides are 9 cm., 12 cm., and 15 cm. Measure the largest angle. What kind of a triangle is it?

10. What do you observe about the ratio of the lengths of the sides of the triangles in examples 6, 8, and 9? Draw all three, using the same point as the vertex of the right angle.

11. Construct a triangle having a side two inches long, and adjacent (adjoining) angles, one 60° and the other 30°. Use the protractor to get these angles.

12. Measure the third angle of your triangle in example 11. How large is it?

13. Measure the longest side and the shortest side of that triangle. What is their ratio?

14. Construct a triangle having two sides of 4 centimeters each and the angle between them a right angle. Construct the right angle with your compasses. Check with your protractor.

15. Measure the acute angles in your triangle of example 14. What do you observe?

16. Draw five triangles of different shapes and sizes. Measure with the protractor each of the angles and fill out a table like the following:

TRIANGLE	1ST ∠	2ND ∠	3RD ∠	SUM OF ANGLES
1				
2				
3				
4				
5				

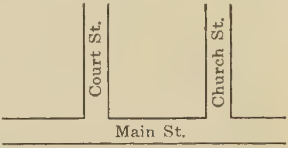
What do you observe about the sum of the angles in each triangle?

We conclude that the sum of the angles of any triangle is —— degrees. If the angles of a triangle are all equal, how many degrees in each angle?

17. What kind of a triangle has its angles all equal? See page 112.

18. Some pupils can explain how to construct an angle of 60° without the use of a protractor. Can you?

How to Construct Parallel Lines. — Look at this map showing part of a city. Court Street and Church Street both make right angles with Main Street. In other words they are both perpendicular to Main Street. If Court Street and Church Street are straight lines, perpendicular to Main Street, will they ever meet? If not, they are ______.

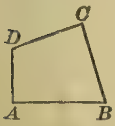


Two straight lines perpendicular to the same straight line are parallel.

If we wish to draw two parallel lines two inches apart, we first draw a straight line AB about three inches long. On this we mark two points M and N two inches apart. At these points we erect perpendiculars to the line AB.

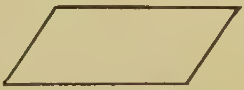
To prove these lines parallel we mark off on each a point four inches from the line AB. We now measure the distance between these points. If our work is accurate, this distance should be two inches. Why?

Quadrilaterals or Figures with Four (quadri) **Sides** (laterals). — A *quadrilateral* is any plane figure that is bounded by four straight lines, as the figure $ABCD$.



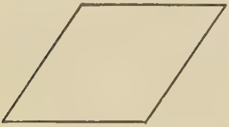
There are several kinds of quadrilaterals.

A *parallelogram* is a quadrilateral in which both pairs of opposite sides are parallel.



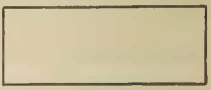
PARALLELOGRAM

Measure the opposite sides of this parallelogram. You will find that they are equal.



Rhombus

A *rhombus* is an oblique parallelogram whose sides are all equal.



Rectangle

A *rectangle* is a parallelogram whose angles are all right angles.

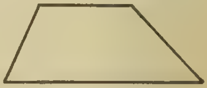
A *square* is a rectangle whose sides are all equal.

Square

Are the opposite sides of a rectangle and of a square parallel?

What is the difference between a square and a rhombus? between a rectangle and a square? Is a square a rectangle? May a rectangle be a square? Must it be?

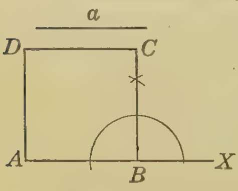
A *trapezoid* is a quadrilateral with two and only two sides parallel.



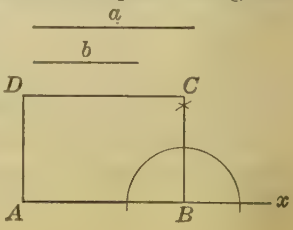
Trapezoid

How to Construct Quadrilaterals. — Study each of the following figures showing how to construct a square with a given side, a rectangle with two given sides, and a parallelogram with two given sides and an angle between them. Can you explain just how it is done?

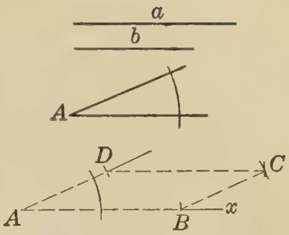
To construct a square having each side equal to a given line *a*.



To construct a rectangle having the sides equal to *a* and *b* respectively.



To construct a parallelogram having two sides equal to *a* and *b* respectively and an angle between them equal to the angle *A*.

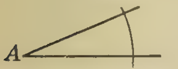


Using Geometry to Construct Quadrilaterals

1. Construct a square each side of which is 5 centimeters.

2. Construct a rectangle 3 inches long and $1\frac{1}{4}$ inches wide.

3. Construct a parallelogram having one side 3 inches long and another 2 inches long and the angle between them equal to angle *A*.



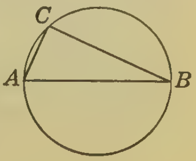
4. Construct a rectangle twice as long as it is wide. Show how to divide it into two equal squares.

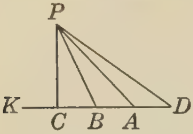
More Systematic Review

1. Construct with your protractor each of the following angles and its complement. *Hint:* See page 83.

Angle	Its Complement
60°	
45°	
70°	
25°	

2. With your compasses construct a circle like the adjoining one, using a radius of 1 inch. Then draw the diameter through the center. Now take any point on the circle such as *C*. Draw *AC* and *BC*. With your protractor measure the angle *ACB*. What kind of an angle is it? What kind of a triangle is *ABC*?





3. Construct from the point P four lines such as shown in the adjoining figure. With your ruler measure each of these lines. Which is the shortest? If the line PC is perpendicular to KD and the other lines are not perpendicular to KD, what would you say about the length of the perpendicular to a line as compared with any other straight line which is not perpendicular to the given line? Now with your protractor measure the angles PCD, PBD, and PAD. What do you find out about the size of these angles?

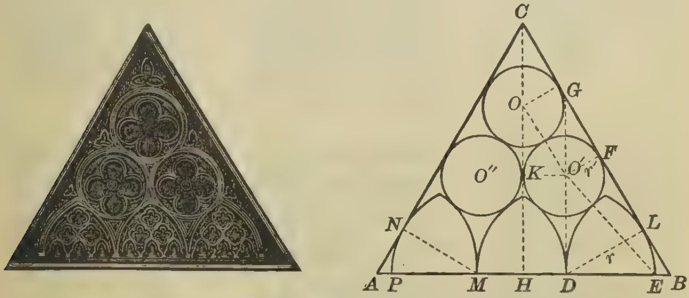
4. Construct a circle with a radius of $1\frac{1}{2}$ inches. Suppose that you now wish to represent a wheel with 12 spokes equally distant apart. How many degrees do you think there will be between any two adjoining spokes? With your protractor and ruler represent the wheel and spokes. Do not forget the hub.

Using Geometry to Construct Designs. — When archi-

The Church of St. Ouen at Rouen

Note the beauty of design through the use of geometric figures.

tects and artists make designs for buildings, tiling, wall paper, linoleum, or rugs, they use the same geometric tools which you have used — lines, circles, angles, triangles, quadrilaterals, perpendiculars, and parallels. Is it not remarkable that they secure so great a variety of designs with so few simple geometric tools? With ruler and compasses you can make almost any design. But of course for variety you must use not only triangles and quadrilaterals, but figures of more than four sides. So you need to learn to construct figures of more than four sides.



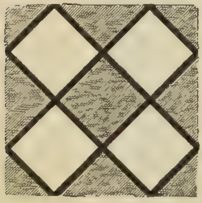
A WINDOW IN WESTMINSTER ABBEY AND THE GEOMETRIC DESIGN FROM WHICH IT WAS MADE

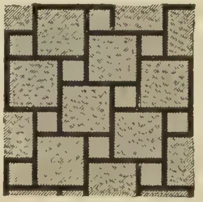
Polygons or Many (poly) **Sided Figures** are plane figures bounded by three or more straight lines. A polygon of three sides is called what? of four sides? The names of other polygons are given in the following table.

NO. OF SIDES	NAME OF POLYGON	NO. OF SIDES	NAME OF POLYGON
5	Pentagon	9	Nonagon
6	Hexagon	10	Decagon
7	Heptagon	12	Dodecagon
8	Octagon	15	Pentadecagon

A *regular* polygon has all its sides equal and all its angles equal. Name a certain kind of triangle which is a regular polygon. What kind of a quadrilateral is a regular polygon?

Using Geometry in Linoleum Designs. — Here are six linoleum patterns. The first three designs are made from different sizes of squares and rectangles.


1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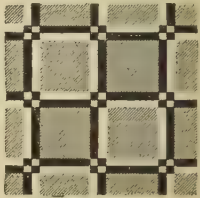
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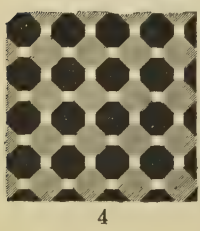
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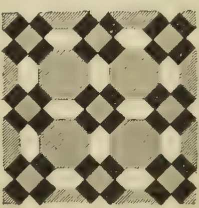
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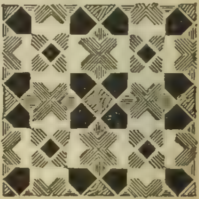
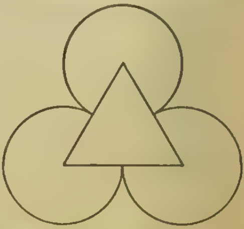

6

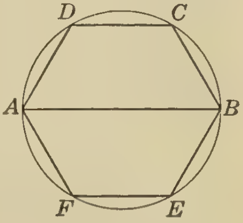
What kinds of polygons are used in number 4? In number 5? Which of the polygons in number 5 are regular? Which are irregular? Can you find irregular pentagons in number 6?

Practice in Designs

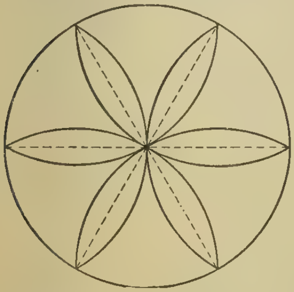


1. Draw an equilateral triangle, each of whose sides shall be 1 inch. Now using each vertex as the center of a circle and one half the length of one of the sides as a radius, construct the parts of each circle as shown in the diagram. The outer figure is known as a *trefoil.* It is often used in architecture.

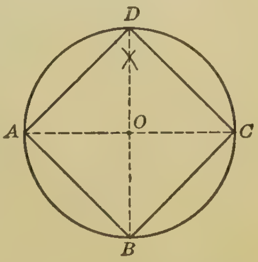
2. Draw a circle with a radius of 1 inch. Without changing your compasses mark off on the circumference arcs beginning at A or B. Join the points A and D, D and C, C and B, B and E, E and F, and F and A with straight lines. You should have a regular hexagon.



3. Using the radius of the circle to mark off equal divisions on the circle, draw the accompanying diagram by the use of your compasses and ruler. Now erase the dotted lines.



4. Draw a circle with a radius of 1 inch. Draw the diameter AC through the center O. Draw another diameter DB through O, perpendicular to AC. Join the ends of both diameters with straight lines as in the figure. The square which you have is said to be *inscribed* within the circle.

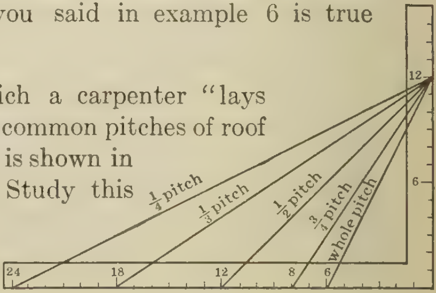


5. Construct a parallelogram with sides of $1\frac{3}{4}$ inches and $2\frac{1}{4}$ inches. Draw the lines joining the opposite corners of the parallelogram. These lines are called *diagonals* of the parallelogram. Measure the distance from their intersection to opposite corners on the *same* diagonal. What do you find? Can you state a rule about the diagonals of a parallelogram?

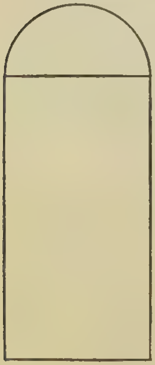
6. Draw a rectangle with sides of $2\frac{1}{2}$ inches and $1\frac{1}{2}$ inches. Draw the diagonals. Measure each of them. What do you find? Then we can say that the diagonals of a rectangle are ——.

7. Construct a square each side of which shall be $2\frac{1}{2}$ inches. Draw the diagonals. Measure each of them. Do you find that what you said in example 6 is true in the square?

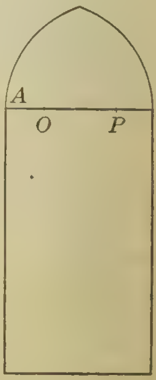
8. The way in which a carpenter "lays off" some of the more common pitches of roof by the use of his square is shown in the adjoining figure. Study this figure and tell all you can about how to "lay off" each kind of a pitch of a roof.



9. Draw a pattern of a Roman[1] window like the accompanying diagram. Construct your rectangle twice as long as it is wide.



10. Draw a pattern of a Gothic[1] window. Construct the rectangle three times as long as the width. Use O and P as centers for the arcs and AP as a radius.



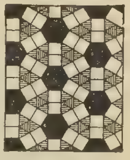
11. Study each of the designs of tiling and parquet flooring, etc., on page 125. See how many figures you can pick out and name.

---

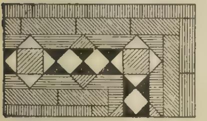
[1] In architecture round arches are called "Roman" and pointed arches "Gothic."

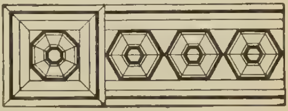
REVIEW TRIAL OF I. R. B. TESTS, CHAPTER IV

Take the third trial of the two tests of Chapter IV. Record your new (and improved) scores in the Review Trial column.



This Review affords you the same opportunity which the preceding chapter did of adding another new feature to your I. R. B. This time (also in out-of-school hours) you can make the very interesting records of your geometric constructions; for example, bisect lines and angles, draw perpendiculars, construct all kinds of angles, triangles, and quadrilaterals, draw parallel lines and geometric designs, etc. Turn back through the pages of this chapter to discover how many interesting constructions you can add to your I. R. B. records.





Word Problems

1. How do you construct a right angle at a point P in a line AB by the use of compasses? How could you check your work?

2. If you knew two of the angles of a triangle, how could you find the third angle?

3. If you were given the temperatures of a hospital patient for each hour, how would you proceed to make a graph of the temperatures for the twenty-four hours of the day?

4. If you knew the divisor, quotient, and remainder, how would you find the dividend?

5. John knew the average sales per day in a store for a month of 26 working days. How could he find the total sales?

6. Henry knew the total sales of a store for the 26 working days of a certain month. How could he find the average sales per day?

7. If you knew the total number of miles that you must make in an automobile trip during your vacation, what other fact would you have to know to find the daily average distance to be covered? How would you find it?

FOURTH TRIAL OF I. R. B. TESTS OF CHAPTERS I AND II

Since Chapter V has no I. R. B. tests, there is no End-of-Chapter Trial. This makes your opportunity to take the Fourth Trial of some of the preceding I. R. B. tests. If you have not already taken these tests for the fourth time, do so for the tests of Chapters I and II. Provided you have worked diligently on your other trials you may now have difficulty in improving the best of the first three scores. Try it, anyway. Enter at least as good a record in the Fourth Trial column.

Review

1. Ruth Malcolm bought $3\frac{3}{4}$ yards of silk ribbon at 25 cents a yard. How much did she pay for it?

2. Max Hoffman bought 3 rolls of films for his camera at 45 cents each. After taking the pictures he had them developed and two prints made of each of the 18 pictures. Developing costs 10 cents a roll and the pictures 8 cents each. Max said he thought two dollars would be enough to pay the bill. Was he right?

3. Try estimating the cost of 2 crates of strawberries if each crate contains 24 quarts and they cost $12\frac{1}{2}$ cents a quart.

4. The average attendance for a week in Helen's class was 24. Monday the attendance was 23, Tuesday 20, Wednesday 25, Thursday 28. How many were present Friday?

5. On a swift mountain stream a motorboat could make $10\frac{1}{2}$ miles an hour with the current, but only $2\frac{1}{2}$ miles an hour against it. The rate of the boat in still water is the average of these two rates. How fast can the boat go in still water? What is the rate of the current?

A Swift Mountain Stream

6. Look up in the encyclopedia the height of the following mountains: Mt. Everest, Mt. Blanc, Mt. McKinley, Mt. Mitchell, Pikes Peak, Mt. Washington, and Mt. Marcy. Make a graph of vertical bars to show the height of these mountains, using one inch to represent 10,000 feet.

7. Draw an obtuse angle. By means of the compasses divide it into four equal angles.

8. What is the ratio of 80 rods to a mile? of 9 inches to 3 feet? of $\frac{2}{3}$ to $\frac{3}{4}$?

9. What will gasoline cost for a motor trip from Maine to California, 4000 miles, if my car averages 16 miles to a gallon of gasoline, and the average price of the gasoline is 21 cents a gallon?

10. Construct a triangle having one side $1\frac{1}{2}$ inches and the adjacent angles 46° and 74°. Measure the third angle and the other two sides. What is the sum of the angles of the triangle? What is the sum of its sides?

11. A man bought a lot for $750. He built a house on it which cost him $9500. His new garage cost him $260. He insured his house and garage at a cost of $13.50 to him. He now sold the property so as to make $1200 on all of the above cost. For how much did he sell the property?

12. How many tons of coal at $13.50 a ton will $162 buy?

13. What is the ratio of 7 to 28? 8 to 32? $6\frac{1}{2}$ to 13? $7\frac{1}{3}$ to 22? $8\frac{1}{4}$ to 33?

14. The Port Authority of New York found that the cost of a box of apples to the consumer was made up as follows:

To the farmer $1.19; to the shipping agent where apples are shipped $.26; to various transporting agencies $.78; to the wholesaler $.43; to the jobber, $.53; to the retailer, $1.90. What is the cost of the box of apples to the consumer?

15. How many degrees in the sum of all the angles of a triangle?

16. Could you draw a triangle with angles of 50°, 60°, and 80°? Why?

17. How long does it take the minute hand of a clock to move through a half circle?

18. A man bought a house for $8000. He rented it for one year at $60 a month. His taxes, repairs, and insurance cost him $220 for the year. How much net rent did he obtain? What was the ratio of his net income to the cost of the house?

Draw a rectangle graph to represent all these items.

19. If an improved road increases the value of the adjoining land \$7.00 per acre, how much will a farm of 200 acres increase in value?

20. John earned 50 cents to-day. This was $\frac{1}{4}$ more than he earned yesterday. How much did he earn yesterday?

21. Mr. Salmon sold his automobile for \$480 after using it for one year. If this was $\frac{1}{5}$ less than he paid for it, what did it cost him?

22. If $\frac{3}{4}$ of all of the fur goods made in the United States in a year were made in New York City, and their value amounted to \$168,617,498, how many dollars' worth were made in the United States?

23. How many kilometers in 425 miles? See appendix tables.

24. A certain family budget was stated as follows:

Housekeeping	\$660.73	Husband's lunches	\$270.41
Shelter	502.52	Wife's pin money	60.13
Household renewals	96.41	Income tax	27.54
Medical and dental	89.02	Clothing, husband	183.41
Vacation and amusements	366.81	Clothing, wife	199.49
		Clothing, boy	96.41
Charitable contributions	59.12	Miscellaneous	199.53
		Savings	552.57

Find the total yearly income of that family.

Review of Fractions

1. $\frac{1}{5}$ of 20 = ?
2. $\frac{1}{6}$ of 42 = ?
3. $\frac{1}{12}$ of 72 = ?
4. $\frac{1}{9}$ of 36 = ?
5. $\frac{1}{8}$ of 144 = ?
6. $\frac{1}{2}$ of $\frac{1}{2}$ = ?
7. $\frac{1}{2}$ of $\frac{1}{3}$ = ?
8. $\frac{1}{4}$ of $\frac{1}{3}$ = ?
9. $\frac{1}{7}$ of $\frac{14}{15}$ = ?
10. $\frac{3}{4}$ of $\frac{8}{9}$ = ?
11. $\frac{5}{6}$ of $\frac{18}{25}$ = ?
12. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ = ?

Review of Measurements

See appendix.

How do you change

in. to ft.?
gal. to qts.?
doz. to gross?
ft. to yd.?
qt. to pk.?
ft. to in.?
sq. yd. to sq. ft.?
bu. to qts.?
rd. to mi.?
oz. to lb.?
doz. to things?
mi. to ft.?
cu. ft. to cu. yd.?
things to gross?
sq. ft. to sq. yd.?

CHAPTER VI

SIMPLIFYING ARITHMETIC BY GEOMETRY AND ALGEBRA

Putting Geometry to Work. — In the last chapter we learned to recognize and construct lines, angles, circles, and other geometric figures. Now we are going to use them to simplify our work in arithmetic. In this way the New Mathematics increases our skill in solving everyday problems.

A Ball Game on a City Playground

Here is another picture of a baseball field. The bases are 90 feet apart and the base lines meet at right angles. This makes a *square* with *dimensions* (measurements) of 90 feet by 90 feet. The total space inclosed is the actual *size* of the figure. Mathematics uses the name *area* in referring to the size of a figure. The *distance* *around* the figure is called the *perim′eter.* (*Peri* means *around*, and *meter* means *measure.*)

What is the perimeter of a baseball diamond?

What will it cost to seed the diamond, if grass seed is 75 cents a pound and one pound seeds 45 square yards?

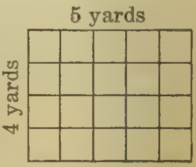
From the preceding illustration we can see how arithmetic and geometry work together to help us. By their aid a farmer can estimate pretty closely just how much wheat he needs to seed a field; the building contractor can accurately determine the floor, wall, and ceiling space of the buildings which he puts up; all of us can from time to time make use of the help which geometry and arithmetic are always ready to offer. Can you name several ways in which you have made use of such help?

Measuring Area. — Area is measured by determining how many square units it contains. These square units may be square miles, square rods, square yards, square feet, square inches, square centimeters, etc. A square inch is a square each side of which is one inch. A square centimeter is a square each side of which is one centimeter. See the tables at the back of the book.

1 sq. cm.	1 sq. cm.	1 sq. cm.
1 sq. cm.	1 sq. cm.	1 sq. cm.

This figure contains 6 square centimeters. What kind of figure is it?

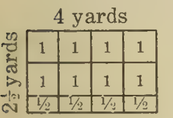
Area of a Rectangle. — Gerald's mother wished to put linoleum on her kitchen floor, and asked Gerald to find out how many square yards it contained. He measured it with a yard stick and found that it was 4 yards wide and 5 yards long.



Gerald drew a diagram to scale like this, using $\frac{1}{4}$ inch to represent a yard. Then he drew lines parallel to the sides and divided the diagram up into squares, each of which represents a square yard. Gerald counted the squares and told his mother that the area of the floor was 20 square yards. Then his mother told him that he had done more work than was necessary. She said to him "If you study your drawing, you will find that there are 4 rows of 5 square yards each, or 4 times 5 square yards equals 20 square yards."

In other words, the number of yards in *length* times the number of yards in *width* equals the number of *square* yards of *area*.

Gerald wished to know whether he could always use this method even if the length or width contained fractions, so he measured the pantry floor which was 4 yards long and $2\frac{1}{2}$ yards wide. He made a drawing like the one above.



How many square yards in this figure?

$$4 \times 2\frac{1}{2} = ?$$

Do these results agree?

We conclude that we can always find the area of any rectangle by the following rule. *To find the number of square units in the area of a rectangle, multiply the number of units in the length by the number of like units in the width.*

A Formula for Area. — When on page 107 we used symbols for angles and arcs, we learned that the New Mathematics is always trying to help us do things in shorter and easier ways. One of the most helpful short-cuts in mathematics is called a *formula*.

A formula is a rule expressed in symbols. If we wish to express the rule above in symbols, we let the symbol l stand for the number of units in the length, the symbol w stand for the number of units in the width, and the symbol A stand for the number of square units in the area. This gives us the formula

$$\boldsymbol{A = lw.}$$

The main thing to remember in formulas is that symbols placed side by side are to be *multiplied*, not added. In this formula lw means l times w.

The length of a rectangle is usually called its *base*, and its width is called its *altitude*. Altitude means the same as height and is usually represented by the letter h. The base is represented by the letter b. So the formula $\boldsymbol{A = lw}$ may also be written

$$\boldsymbol{A = bh.}$$

Illustrative Example. Find the area of a rectangle 8 inches long and 5 inches wide.

Solution:

$$\begin{aligned} A &= lw \ (l \text{ times } w) \\ &= 8 \text{ times } 5 \\ &= 40 \text{ square inches.} \end{aligned}$$

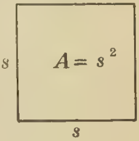
Note that we substitute 8 for l and 5 for w, just as one player is substituted for another in football.

The Formula Brings Us to Algebra. — In our formulas we saw that A was a symbol for *all* areas. The letter l is a symbol for *any* length. This use of letters brings us to another branch of the New Mathematics which uses letters as well as figures to represent numbers. This branch of mathematics we call *Algebra*.

Algebra uses the processes of arithmetic besides some special ones of its own. It uses the same signs:

$+, -, \times, \div$. The most important thing to remember in algebra, as in the case of formulas, is that two letters or letters and figures, when written side by side, are to be *multiplied*. Thus bh means b times h; $2\,l$ means 2 times l.

Squares in Algebra. — The use of squares is very common in algebra. There is a simple formula for getting the area of a square. If s represents the number of units in the side of the square, then $A = s \times s$. We do not usually write s times s as ss but write it s^2 instead. This gives us the formula



$$A = s^2.$$

This is read "A equals s square."

Here the 2 is called an *exponent*. It is a small figure written above and at the right of a letter or number to show how many times the letter or number is to be used as a factor. Here it means that s is multiplied by s, or 2 s's are multiplied together.

How much is 2^2? 3^2? 5^2? 6^2? 8^2? 10^2?

Illustrative Example. Find the area of a square whose side is 15 inches.

Solution:
$$\begin{aligned} A &= s^2 \\ &= 15^2 \\ &= 15 \times 15 \\ &= 225 \text{ square inches.} \end{aligned}$$

Using Algebra in Perimeter Problems. — If the sides of a figure are straight-line segments, we may find the perimeter by adding the lengths of all the sides, as in these illustrations.

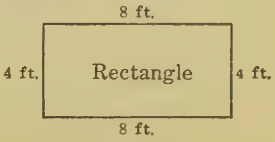


Figure 1

Perimeter of Figure 1

$= (8 + 4 + 8 + 4)$ ft. $= 24$ ft.

Perimeter of Figure 2

$= (l + w + l + w)$ ft.

$= (2\,l + 2\,w)$ ft.

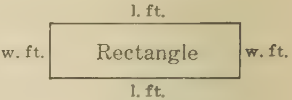


Figure 2

Perimeter of Figure 3

$= (6 + 6 + 6 + 6)$ ft. $= 4 \times 6$ ft.

$= 24$ ft.

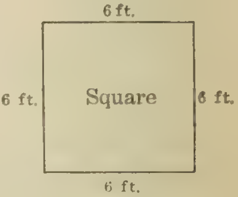


Figure 3

Perimeter of Figure 4

$= (s + s + s + s)$ ft. $= 4\,s$ ft.

Notice that we do not have to write $4 \times s$ as in the line above we wrote 4×6.

I. R. B. Test [**1**]. Prepare a chart for Chapter VI (three tests). Enter First Trial scores. Use examples 1 10 for this test. Let the Steering Committee determine the standard time.

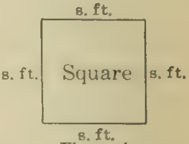


Figure 4

DO-IT-IN-YOUR-HEAD PLUS PENCIL-PAPER PRACTICE

Divide the class into three teams. Let the first and second teams compete in an oral contest, members of each team alternating in giving the answers to the following 16 examples.

Let the third team check the answers. In case of a wrong answer let the member of the third team who first gives the right answer replace the pupil of either the first or second team who fails. Score one for each correct answer given by the first and second teams; score two to the credit of the third team for each member replacing a member of either the first or second team.

In case an answer cannot be given orally let the solution be given on the blackboard. Your teacher and Steering Committee will make additional examples, if necessary, to complete the contest.

Area Exercise

Find the areas of the following rectangles:

1. Length 7 feet, width 4 feet.

2. Length 6 inches, width 4 inches.

3. Length 18 rods, width $6\frac{1}{3}$ rods.

4. Length $8\frac{1}{2}$ feet, width $5\frac{3}{5}$ feet.

Find the area of the following squares:

5. Length of one side, 12 inches.

6. One side is $5\frac{1}{2}$ yards.

7. One side is $7\frac{3}{10}$ inches.

8. What are the perimeters of each of the squares in examples 5, 6, and 7?

9. Find the number of square feet in a rectangular garden 20 feet wide and 35 feet long.

10. How long must a fence be to inclose the garden in example 9?

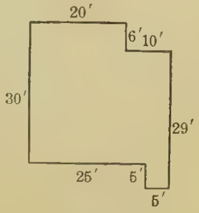
11. Give the table of square measure in the Appendix of the book.

12. Measure the top of your desk. How many square inches does it contain? How many square feet? What is its perimeter?

13. A small farm was in the shape of a rectangle. It was 45 rods long and 22 rods wide. How many acres did it contain?

14. How long a fence will be required to inclose it? What part of a mile is this?

15. The ground floor of a house had the shape and dimensions shown in the diagram. How many square feet of floor space were there? *Hint:* Draw lines to divide the floor into rectangles and a square.



A HARVARD YALE GAME IN THE YALE BOWL

16. A football field is a rectangle 300 feet long and 160 feet wide. How much lime will it take to mark its perimeter, using a pound of lime for each four feet? Draw a plan of the field by letting 1 inch represent 60 feet. Then draw parallel lines to the end lines to represent the lines which are 5 yards apart and generally spoken of as the 5-yard lines.

The Habit of Systematic Review

1. The number of 8th grade graduates in the state of Illinois in 1913 was 41,361. The number of graduates in 1923 was 70,698. Find the increase and the ratio of the graduates in 1913 to those in 1923.

2. The average number of persons employed in agriculture for every 1000 acres of land is

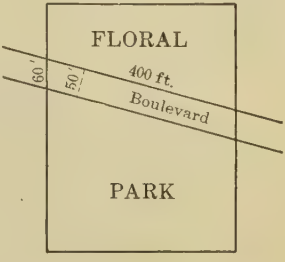
Italy . . .	235	France . . .	120	United States .	41
Germany . .	160	Eng. and Wales .	105		

How many more are employed per thousand acres in each of the other countries than in the United States? Draw bar graphs to represent the above figures.

3. If English wages are about two fifths of American wages, what would be the English wage for an American receiving $2000? What would be the American wage for an Englishman receiving $1200?

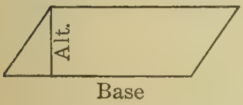
4. How many square rods in a square mile?

Finding the Area of a Parallelogram by Algebraic Formula. — A boulevard 50 feet wide was laid out through Floral Park as shown in the drawing. That part of the boulevard within the park was to be paved with brick and the rest with concrete. What shape is the part to be paved with brick? What is its length? its width?



How long is the short side of the parallelogram? Is this the same as the width?

Call the longest side the *base*. The width is measured on a line perpendicular to the base. This line is called the *altitude* of the parallelogram. The altitude is really the *distance* between the parallel lines.



Draw two altitudes as shown in Figure 1. What kind of an angle is $\angle BCD$? If you are not sure, use your protractor.

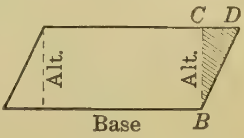


Figure 1

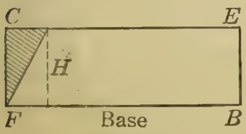


Figure 2

Cut off the shaded triangle and place it in the position shown in Figure 2. What kind of an angle have you at C? at B? at F? at E?

What kind of a figure is now formed? Is the area the same as it was in Figure 1? Is the base the same as before? Is the altitude the same as before? Now that it is a rectangle we know that $A = bh$. Hence the area of a parallelogram is also found by the formula $A = bh$. We are now able to find the number of square feet to be paved with brick in Floral Park Boulevard, for $b = 400$ feet, $h = 50$ feet.

Therefore
$$\begin{aligned} A &= bh \\ &= 400 \times 50 \\ &= 20{,}000 \text{ square feet.} \end{aligned}$$

I. R. B. Test [2]. Enter First Trial scores for time and accuracy in solving the following ten problems. Let the Steering Committee and your teacher determine the standard time.

Parallelogram Practice

Find the area of the following parallelograms:

1. Base 7 feet, altitude 3 feet.

2. Base $6\frac{1}{2}$ inches, altitude 4 inches.

3. Base $8\frac{1}{2}$ feet, altitude 6 feet.

4. Find the area in square feet of a parallelogram when $b = 8$ yards and $h = 10$ feet. What is the area in square yards?

5. Find the area of a parallelogram whose base is $16' \, 3''$ (16 ft. 3 in.) and whose altitude is $4' \, 6''$.

6. What is the area of a parallelogram whose altitude is 8 feet and whose base is $\frac{1}{4}$ larger than the altitude?

7. Find the area of a parallelogram if the base is 12 feet and the altitude is 3 feet.

8. What is the area of a parallelogram whose base is 6 feet and whose altitude is 6 feet?

9. The area of a parallelogram is 24 square feet. The base is 6 feet. What is the altitude?

10. If the altitude is 7 inches and the area is $84\frac{7}{10}$ square inches, what is the base of the parallelogram?

A System Systematically Worked

1. A part of a floor in the form of a rectangle 12 feet by 10 feet is to be tiled with tiles 8 inches square. How many tiles will it take?

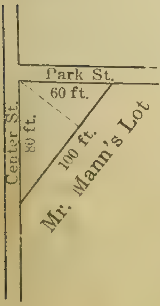
2. A rectangular farm is 320 rods long and 200 rods wide. How many acres does it contain? What will it cost at $20 per acre?

3. A square piece of land is $12\frac{7}{10}$ rods on a side. How many feet of fence will it require to build a fence around it? What is your formula for this?

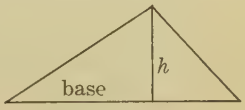
4. At $1.20 a square yard what will it cost to put down a concrete walk 120 feet by $4\frac{1}{2}$ feet?

5. Owing to poor business conditions in a certain town a man had to sell for $10,000 a house which cost him $12,500. What was the ratio of the selling price to the cost?

6. One boy estimated that $8\frac{1}{2} \times 6\frac{1}{2} = 60\frac{3}{4}$. Another boy said that $50\frac{1}{2}$ was nearer the correct result. A third boy said that 54 was nearest correct. Which one do you think estimated the best? How could you estimate the correct result in this example rather closely?

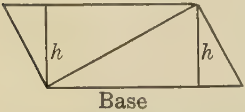


Finding the Area of a Triangle by Formula. — In a triangle one side is called the *base*, and the perpendicular line drawn from the opposite vertex to the base is called the *altitude* or height of the triangle.



Harold Mann's father owns a triangular lot at the corner of Park and Center streets. It has the dimensions shown in the figure. He told Harold that he would give him a dollar if he could tell him how many square feet were in the lot. Harold asked his teacher, and she gave him this experiment to try. Let each one of the class try it.

Draw a triangle. Cut it out of the paper. Now use it as a pattern and cut out another exactly like it. Draw the altitude of each. Now place them together as shown in the figure.

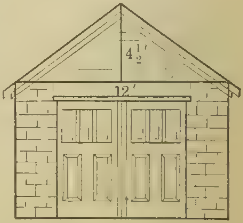


What kind of figure is made by the two triangles? Is its base the same as that of the triangle? Is its altitude the same? The area of the parallelogram is $b \times h$. The triangle is what part of the parallelogram? Then the area of a triangle must be $\frac{1}{2}$ its base times its altitude. Harold rightly concluded that the area of a triangle can always be expressed by the formula

$$A = \tfrac{1}{2}bh.$$

That afternoon Harold went home and measured the dotted line shown in the figure, being very careful to get it perpendicular to the 100-foot base. He found it to be 48 feet long. "Then," he said, "the area of the triangle is

$\frac{1}{\cancel{2}} \times 100 \times \overset{24}{\cancel{48}}$ or 2400 square feet."



Do you think he earned the dollar?

Jack Wood, when he learned about this experiment, went home and measured the gable to his father's garage. He found it was 12 feet long and $4\frac{1}{2}$ feet high. $4\frac{1}{2}$ feet $= \frac{9}{2}$ feet.

Then he found the area was $\frac{1}{\cancel{2}} \times \overset{3}{\cancel{12}} \times \frac{9}{\cancel{2}}$, or 27 square feet.

How much will it cost to paint the two gables of the garage, each two coats of paint, if each coat costs 30 cents a square yard?

An Attractive Garage

Using Geometry and Algebra to Simplify Arithmetic

Work the following mentally wherever you can. Use algebra and geometry as well as arithmetic.

1. What is the perimeter of Mr. Mann's lot?

Find the area of each of the following triangles:

2. Base 14 yards, height 6 yards.

3. Base 24 feet, height 16 feet.

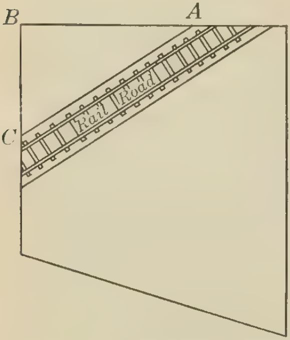
4. Base $12\frac{1}{2}$ inches, height 6 inches.

5. Base $18\frac{1}{2}$ inches, height $10\frac{1}{2}$ inches.

6. Base $20\frac{2}{3}$ feet, height $5\frac{4}{5}$ feet.

7. Base 3 yards, height 7 feet.

8. Base $2\frac{1}{2}$ feet, height 14 inches.



9. The straight tracks of the Pennsylvania Railway cross Mr. Slate's farm so as to cut off a lot ABC, which is a triangle. The angle at B is a right angle. If AB is called the base, what line is the altitude? AB is 16 rods and $BC = 10$ rods. What is the area of this lot? What part of an acre is it?

10. Three streets inclose a triangular lot whose sides are 60 feet, 70 feet, and 85 feet. Using 1 inch to represent 20 feet, construct this triangle to scale. Construct its altitude. Measure it carefully. Find then as closely as you can the area of this lot.

Practice Makes Perfect

1. A grass plot is in the form of an equilateral triangle. The perimeter is 360 feet. What is the length of one side?

2. My lot is a corner lot 130 feet on one street and 90 feet on another street. The two streets are at right angles to each other. If I build a concrete walk 4 feet wide on both sides of it, what will it cost me at 20 cents per square foot?

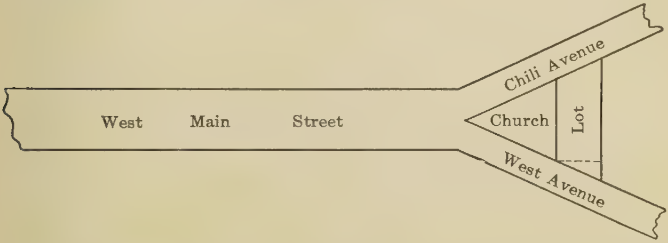
3. Members of a certain church sold their very valuable corner property. They had asked $350 a square foot for it. If the lot was in the form of a rectangle 120 feet on the avenue and 84 feet on the street, what would it sell for at that price?

4. Dick, Whitey, and Tom were given the following examples:

Add: $45, $71.25, $94.50, $83, $162.50, $75.40, and $83.25. Dick got $567.87 for his answer. Whitey got $615.10. Tom said the sum was $714.90. None of them had it right.

Which do you think, without adding these numbers yourself, had it nearly correct? That is, estimate your answer.

Algebra and Geometry Coöperate to Find the Area of a Trapezoid. — In the city of Rochester there are two streets which intersect like this. The lot behind the church is a trapezoid. Which sides are parallel? Which sides are not parallel?



In a trapezoid the altitude is a perpendicular line drawn between the parallel sides like the dotted line in the figure. The parallel sides are both called *bases*. To distinguish them we shall mark one of them b and the other b' (read " b prime "). Let each member of the class try this experiment.

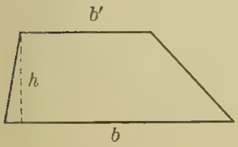


Figure 1

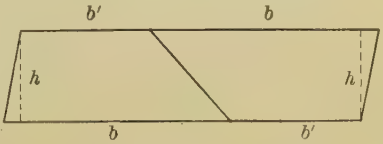


Figure 2

Construct a trapezoid like Figure 1. Cut it out of the paper. Then using it as a pattern, cut out another exactly like it. Place them together as shown in Figure 2. The two trapezoids together make a parallelogram. The base of this parallelogram is $(b + b')$. The altitude of the parallelogram is h, the same as that of the trapezoid. How do we find the area of a parallelogram? Then the area of this parallelogram must be $(b + b')h$.

The trapezoid (Figure 1) is what part of the parallelogram (Figure 2)? Then the area of the trapezoid must be $\frac{1}{2}(b + b')h$.

Illustrative Example. Find the area of the Rochester lot shown on page 145 if one base is 180 feet, the other 210 feet, and the altitude 45 feet.

Solution: $b + b' = 390$

$$A = \tfrac{1}{2} \times 390 \times 45 = 8775 \text{ square feet.}$$

Estimate, Work, and Check

Find the area of the following trapezoids:

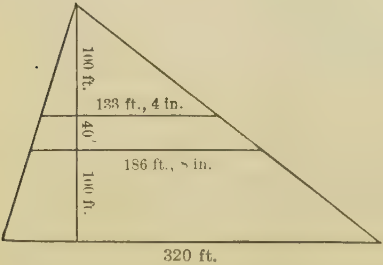
1. $b = 64$ feet, $b' = 48$ feet, $h = 16$ feet.

2. $b = 32$ rods, $b' = 18$ rods, $h = 6$ rods.

3. $b = 8$ yards, $b' = 5$ yards, $h = 3$ yards.

4. $b = 60$ feet, $b' = 36$ feet, $h = 80$ feet.

5. $b = 19$ inches, $b' = 11$ inches, $h = 15$ inches.



6. Mr. Richards owns a triangular lot, through which the city ran a street, with dimensions as shown here.

(*a*) Find the area of the lot above the street.

(*b*) Find the area of the street.

(*c*) Find the area of the lot below the street.

A Brief Test of the Review Habit

1. A paved street is 1 mile long and 36 feet wide. How many paving blocks did it require if each block was 8 inches by 4 inches?

2. At $3\frac{1}{4}$ cents a mile how much would it cost to travel 200 kilometers?

3. My new lawn is 120 feet by 25 feet. What will it cost me to have it sodded at 8 cents per square foot?

4. What part of a square inch is a square $\frac{1}{2}$ of an inch on a side?

5. Define

(*a*) A parallelogram.
(*b*) A rhombus.
(*c*) A rectangle.
(*d*) A square.

Finding the Circumference of a Circle. An Interesting Experiment. — The distance around a circle, or its length, is called the *circumference.*

Try the following experiment. Collect several circular objects such as a half dollar, a saucer, or an ink bottle. Measure carefully the diameter of each. Cut a narrow strip of paper and wrap it around the edge of the circular object you measure so that the edges of the paper just come together. Measure the length of this strip. This represents the circumference of the object. Measure each of the objects in the same way.

To check your work, draw a line on a sheet of paper. Now mark a point on the edge of your object, for instance the saucer. Place this point at one end of your line and roll the saucer along the line, being careful not to let it slip. When it has rolled around once, mark a dot on your line where the point on the saucer falls. Now measure the distance between this dot and the end of your line.

As a matter of fact, very careful measurements, as well as other methods of proof which you will study later, have shown that the circumference of any circle is always exactly the same number of times the diameter.

Alvin Ladd of the Roosevelt school got the results shown in the following table.

OBJECT	DIAMETER	CIRCUMFERENCE	$C \div D$
Half dollar	$1\frac{3}{16}''$	$3\frac{3}{4}''$	$3\frac{15}{100}$
Elec. battery	$2\frac{5}{8}''$	$8\frac{1}{4}''$	$3\frac{14}{100}$
Saucer	$6''$	$18\frac{7}{8}''$	$3\frac{14}{100}$
Piano stool	$14\frac{1}{2}''$	$45\frac{1}{2}''$	$3\frac{14}{100}$
Tin can	$4''$	$12\frac{1}{2}''$	$3\frac{12}{100}$
Ink bottle	$2''$	$6\frac{1}{4}''$	$3\frac{12}{100}$

We see that the circumference (C) is almost exactly the same number of times the diameter (D) in every case. Your own experiments will show a similar result if you are careful.

The circumference of a circle is about $3\frac{14}{100}$ or $3\frac{1}{7}$ or $\frac{22}{7}$ times the diameter.

This number is usually represented in formulas by the Greek letter π (pronounced pī).

To find the circumference of a circle, multiply its diameter by π ($\frac{22}{7}$ or 3.14).

In formula form this rule may be stated

$$C = \pi d.$$

The diameter is equal to twice the radius; that is,

$$d = 2r.$$

Therefore if we wish, we may write the formula

$$C = 2\pi r.$$

I. R. B. Test [**3**]. Write merely the answers as you get them in your head, so far as you can. Then use paper and pencil. Finally work all the following fifteen examples with pencil and paper and compare with your first answers. Record the scores of your work in your I. R. B., First Trial column. Let a committee set a standard time.

Find the circumference of the following circles:

1. $d = 4$ feet.
2. $d = 3$ inches.
3. $r = 2$ inches.
4. $r = 4$ inches.
5. $r = 6$ inches.
6. $r = 8$ inches.
7. $d = 15$ rods.
8. $r = 10$ feet.
9. $d = 4\frac{1}{2}$ inches.
10. $r = 5\frac{2}{5}$ feet.

11. What is the circumference of a 28-inch (meaning a diameter of 28 inches) bicycle wheel?

12. How far along the road will that wheel go in one turn? How many turns will it make in going $\frac{1}{4}$ of a mile? $\frac{2}{3}$ of a mile?

13. What is the circumference of a 34-inch automobile wheel? How many turns would it make in going 1 mile? in going 100 miles?

14. If you knew the circumference of a wheel were 176 inches, what would be its diameter?

15. What is the diameter of a circular object whose circumference is 48π? 24π? 12π? 6π?

Systematic Review by the Estimate Plan

Read the examples carefully, think, estimate, work, and check. If you fail in a review example, turn back to the chapter where similar examples are given and try several until you are sure that you can do them correctly.

1. A cubic foot of water weighs $62\frac{1}{2}$ pounds. If a gallon is about $\frac{2}{15}$ of a cubic foot, how much does a gallon of water weigh?

2. Copper is $4\frac{4}{5}$ times as heavy as water. How much will 1 cubic foot of copper weigh?

3. A man receives 5 cents for making a certain article. How many pieces would he have to make to earn $6.00 in one day?

4. A bushel of potatoes weighs 60 pounds. How many bushels can Mr. Malcolm carry in his ton truck without overloading?

5. Jack's father gets an average distance of 15 miles from a gallon of gasoline with his new car. How much gasoline will be needed to tour across the continent (3200 miles)? How much will it cost him at an average cost of 21 cents a gallon?

6. How much will it cost to take the above trip by rail at $3\frac{3}{5}$ cents a mile?

7. If the average total cost of running the above automobile is 10 cents a mile, how much would it cost each passenger to take the above trip by an automobile which carries four passengers? By rail at 3.6 cents per mile? Which is the cheaper and how much?

8. Mr. Malcolm earns $60 a week, working 48 weeks a year, and gets an annual bonus of $120. Mr. Henderson receives a monthly salary of $300 and does not get a bonus. State the ratio of Mr. Malcolm's annual income to that of Mr. Henderson.

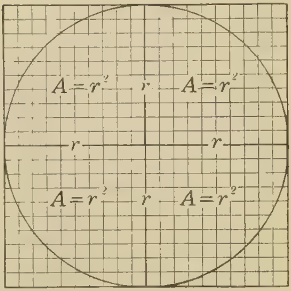
9. West High School has 1700 pupils and 85 teachers. What is the average number of pupils per teacher? If the average annual salary for ten heads of departments is $3600 each and for the 75 other teachers $2500 each, how much money is paid the teachers of this school each year? What is the average annual cost of instruction per pupil in this school?

10. If the salaries of the teachers is $\frac{2}{5}$ of the cost of running the school, how much does it cost to maintain that school for one year? What is the average cost per pupil?

11. A grocer sells eggs at 36 cents a dozen and buys them at $\frac{5}{6}$ of the selling price. What was the cost to him of 12 crates, containing 30 dozen each?

12. In one such shipment 3 dozen eggs were broken. How much did he make on the 12 crates if sold at 36 cents per dozen?

Finding the Area of a Circle. An Interesting Area Experiment. – Draw a circle on graph paper with a radius of one large square (or ten small squares). Draw two diameters perpendicular to each other. Now draw a large square around the circle, as shown in the diagram. The diameters divide this square into four smaller squares. Each side of one of these squares is the radius of the circle, hence the area of each square is r^2.



Therefore the area of the large square drawn around the circle must be $4\ r^2$.

Is the circle as large as this square?

Then the area of the circle must be less than $4\ r^2$.

To get a clearer idea let us count the number of small squares in r^2. There are just 100 of them. Now let us count the number in the circle. We find just 284 whole ones and 60 parts of squares. These 60 part squares may be estimated to average half a square each, so that they are equal to 30 whole squares. The circle then contains about 314 small squares and r^2 contains 100 of them. How many r^2 is the circle? We find then that the circle is equal to about $3.14\ r^2$. What did we call 3.14 in the last section? To what common fraction is it about equal?

100)314(3.14
300
140
100
400
400

This shows us that the formula for the area of a circle is

$$A = \pi r^2.$$

This formula means that when we wish to find the area of a circle, we multiply the radius by itself and the result by π ($\frac{22}{7}$ or 3.14).

Illustrative Example. Find the area of a circle whose radius is 14 inches.

Solution: $A = \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \times 14 = 616$ square inches.

Individual Practice

Take the following test by yourself. Keep a record of your time. Find the circumferences and areas of the circles with the following radii:

1. $r = 3$

2. $r = 10$.

3. $r = 7$.

4. $r = 16$ feet.

5. $r = 6$ rods.

6. $r = 3\frac{1}{2}$ rods.

7. $r = 2$ inches.

8. $r = 4$ inches.

9. $r = 8$ inches.

10. Compare the radii, circumferences, and areas of examples 7, 8, and 9, writing them as below:

Radii *Circumference* *Area*

What do you observe about the circumferences and areas of the above circles as compared with their radii?

11. A cow was tied to a tree with a rope 100 feet long. Over how many square feet could she graze? What part of an acre is this?

12. One of the giant Redwood trees of California has a diameter of 30 feet. What is its circumference?

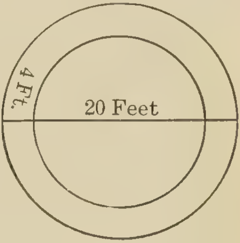
13. If this tree were cut off so that the top of the stump were level, what would be the area of the top of the stump?

14. Henry measured the distance around a telephone pole and found that it was 30 inches. What is the diameter of the pole? *Hint:* The diameter = the circumference divided by π, or $d = \frac{c}{\pi}$.

15. What is the area of a circle if the radius is $6\frac{1}{2}$ inches?

16. A piece of land is in the form of a circle. The diameter of the circle is 20 feet. A walk 4 feet wide is laid around it as shown in the figure.

Find (1) The area of the large circle.
(2) The area of the small circle.
(3) The area of the walk.
(4) The cost of the walk at \$1.80 per square yard.

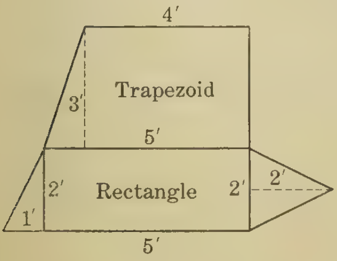


Record of the Algebraic Formulas of Chapter VI

I. R. B. Turn back through the pages of this chapter and you will find a number of formulas which you now know are very valuable aids in your work of the New Mathematics. Copying these Algebraic Formulas into the records of your I. R. B. will help you to memorize them.

Individual Review

1. Find the whole area of the accompanying figure.

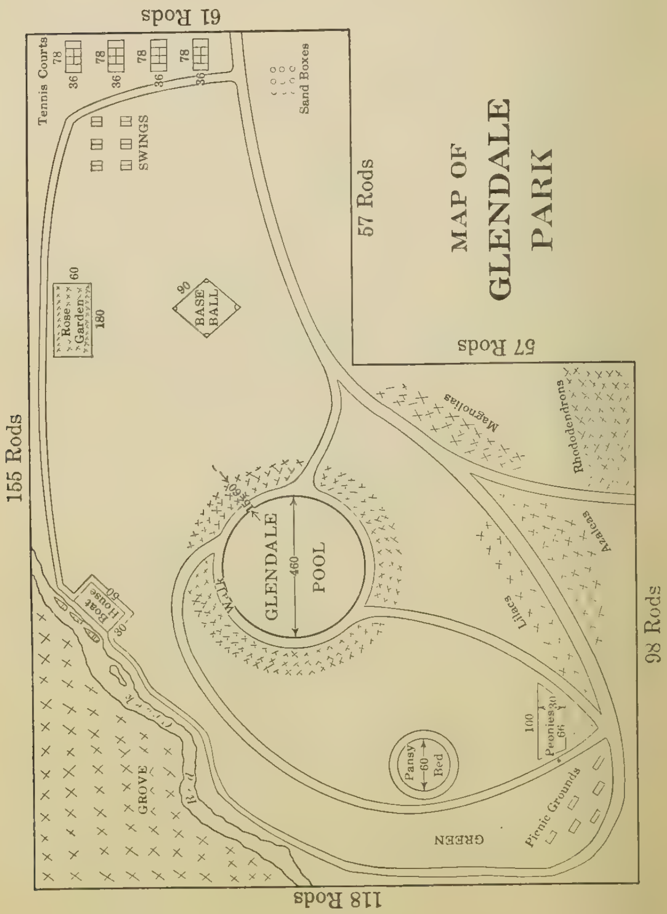


2. A piece of machinery costing \$9600 lessened in value $\frac{1}{12}$ during the first year. What was it worth at the end of the first year?

3. John said that a meter was about $\frac{1}{10}$ more than a yard. Henry said that it was about $\frac{1}{12}$ more than a yard. Which was the nearer correct and how near?

4. What is the area of a triangle whose base is $7\frac{1}{2}$ inches and whose altitude is $3\frac{1}{2}$ inches?

5. A trapezoid has one base of $6\frac{1}{2}$ inches and another base of $4\frac{1}{2}$ inches and an altitude of $2\frac{1}{4}$ inches. What is its area?


MAP OF
GLENDALE
PARK
61 Rods
Tennis Courts
78
36
78
36
78
36
78
36
Sand Boxes
SWINGS
57 Rods
57 Rods
155 Rods
Rose Garden
60
180
BASE BALL
90
Magnolias
Rhododendrons
Azaleas
Lilacs
GLENDALE
460
POOL
Walk
5
60'
Boat House
60
30
GROVE
Road
Pansy
60
Bed
Peonies
30
100
66
Picnic Grounds
GREEN
118 Rods
98 Rods

A PROJECT — LAYING OUT A PUBLIC PARK

A wealthy resident of a certain city gave a fine tract of land in the edge of the city to be used as a public park and playground. It is called Glendale Park. The map shows its shape and dimensions.

The commissioner of parks and his assistants decided to build a circular lagoon or pool in the center of the park in a natural hollow. This pool has a diameter of 460 feet. Around it is a gravel walk 15 feet wide, and around this a border of shrubs and flowers on the sloping ground. This border is 60 feet wide. The stream of water flowing through the park is Red Creek. The corner beyond the creek is covered with fine evergreens.

A boat house 30 by 50 feet was built beside the creek and 12 boats provided for the use of visitors. In the northeast corner four tennis courts were laid out and back of them 6 swings and 10 circular sand boxes were provided for the children. A baseball diamond was also laid out. At the opposite end of the park picnic grounds with tables were provided. A rose garden, pansy bed, and shrubbery were planted as shown, and walks were laid out as indicated on the map.

1. All the angles of the park are right angles. How many acres does it contain?

2. When it was given to the city, this land was worth $1250 an acre. What was the value of the gift?

3. The diameter of the pool is 460 feet. How many acres does it contain?

4. What is the length of the inside of the walk surrounding it?

5. What is the length of the outside of the walk?

6. What is the area of this walk?

7. What is the area including the border of shrubs?

8. Each tennis court is a rectangle 78 by 36 feet. Find the total area of all of them.

9. The baseball diamond is a square 90 feet on a side. What is its area?

10. How far must a player run who makes a two-base hit?

11. The rose garden is a rectangle 180 feet by 60 feet. What is its area?

12. There are 5 rose bushes to each square yard of the garden. How many rose bushes are there?

13. The pansy bed is a circle 60 feet in diameter. What is its area? its circumference?

14. The peony bed is a trapezoid, with bases 100 feet and 66 feet. It is 30 feet between the bases. What is its area?

15. The evergreen grove is somewhat like a triangle. The map is drawn approximately to scale. Measure the base and altitude, find out how many rods each represents; then estimate the area of the grove.

16. Find the area of the ground floor of the boat house.

17. The total length of the walks is $1\frac{4}{5}$ miles, and their average width is 8 feet. Find the area of the walks.

18. Find the cost of the rose bushes at 40 cents each.

19. In preparing the pool 1600 loads of earth were removed at a cost of $1.25 a load. Find the cost of excavating for the pool.

20. The sand boxes were 12 feet across. Find the area of each. Find the area of all of them.

21. The cost of building the boat house was $10,000; of the swings $15 each; of the sand boxes $12 each; of the picnic tables $10 each; of the boats $30 each. The cost of the flowers and shrubs was $12,820. The labor (laying out

A Beautiful Public Park

This is McKinley Park, Sacramento.

walks, grading, planting shrubs, etc.) was $8720.40. Find the total cost to the city of preparing the park.

22. The cost of caring for the park each year includes the following items:

Caretaker at boat house	$1200.00
Mowing lawns and caring for shrubs	2270.50
Guard at pool	1500.00
Caretaker at playground	1250.00
Water	680.00
Repairs and miscellaneous	175.60
Police protection	2250.00
Lighting	600.00

What does it cost the city to maintain this park each year?

FOURTH TRIAL OF I. R. B. TESTS OF CHAPTERS III AND IV

Unless you have already taken the Fourth Trial of the I. R. B. Tests of preceding chapters, it will now be advisable to complete the trials of some chapters. Look over the pages of Chapters III and IV and take the I. R. B. tests. Enter your scores in the Fourth Trial column. If you cannot surpass a former score, after trying to do so, enter at least an equal score.

Short Tests of Your Review System

1. Find the area of a triangle whose base is 4 feet and whose altitude is 18 inches.

2. How many liquid quarts will a tank of 875 cubic inches capacity hold? *Hint:* One gallon contains 231 cubic inches.

3. $5\frac{1}{2}$ is what part of $16\frac{1}{2}$?

4. If $1\frac{1}{4}$ cu. ft. contain 1 bu. (approx.), how many bushels will 20 cubic feet contain?

5. The area of a parallelogram is $31\frac{1}{2}$ square feet. If the altitude is $4\frac{1}{2}$ feet, what is the base?

6. A rectangle is 4 meters long and 13 decimeters wide. How many square dm. in its area?

7. How do you change

bu. to pints?
in. to m.?
mi. to rods?
liters to quarts?
gross to doz.?
pk. to bu.?
pounds to kg.?
cu. in. to cu. ft.?
qt. to gal.?

8. In a factory the men were paid 22 cents a piece for making a certain article. Upon what would their daily wage depend?

9. A man pays $60 a month for rent. If his yearly income is $3600, what is the ratio of his yearly rent to his yearly income? Construct two bar graphs to show this fact.

10. The population of a certain city was 267,779 in 1910. If the population in 1920 was 298,103, what was the increase? What was the average increase per year?

Algebra Coöperating with Geometry and Arithmetic

When you have written the word-answers to the following word-problems, try to substitute letters for the word-answers in as many as you can. Use the letters which seem to you most appropriate.

1. If you have the base and altitude of a rectangle, how do you find the area?

2. Given the area of a parallelogram and the base, how do you find the altitude?

3. If you knew the area and the altitude of a triangle, how could you find the base?

4. If you knew the scale of miles per inch on a certain map, how could you find the distance between any two places on that map?

5. Given the value of an automobile and the cost per thousand dollars to insure it, how would you find the cost to insure that automobile?

6. Given the length and breadth of a rectangle in rods, how would you find the number of acres in it?

7. How would you find the value in American money of a certain number of English pounds? See Appendix.

8. If you knew the area of a trapezoid and the two bases, how would you find the altitude?

Exercise

Copy and complete the following:

RECTANGLES

LENGTH	WIDTH	AREA
17 in.	15 in.	
$8\frac{1}{2}$ in.	6 in.	
14 in.	$6\frac{1}{5}$ in.	
19 in.	$7\frac{1}{3}$ in.	
$15\frac{1}{8}$ in.	$8\frac{1}{3}$ in.	

TRIANGLES

BASE	ALTITUDE	AREA
8 in.	6 in.	
$5\frac{3}{5}$ in.	$3\frac{2}{5}$ in.	
$18\frac{3}{5}$ ft.	$12\frac{1}{2}$ ft.	
$14\frac{1}{2}$ rd.	$6\frac{1}{7}$ rd.	
$16\frac{2}{5}$ ft.	$6\frac{1}{2}$ ft.	

RECTANGLES

AREA	BASE	ALTITUDE
36 sq. ft.	9 ft.	
63 sq. in.		9 in.
54 sq. in.		3 in.
24¾ sq. ft.		3 ft.
57 sq. yd.	8½ yd	

RECTANGLES

LENGTH	WIDTH	PERIMETER
12 in.	6 in.	
9½ in.	3½ in.	
6½ ft.	4½ ft.	
15½ rd.	10¾ rd.	
16½ in.	12¼ in.	

COMMITTEES OF FOUR

To work four sets of area and perimeter problems

Have your Steering Committee, with your teacher as ex-officio member, divide your class into Committees of Four and appoint one chairman for each committee. The chairman of each committee should copy the first set of problems and have each of his committee members copy one of the other three sets. Then on a separate sheet each member, including the chairman, should write the answers to the problems of the set copied. Next the chairman should exchange among his committee the copies of the four sets until each member has written on his separate sheet all 20 answers.

The different chairmen should then exchange answer papers and distribute the exchanged papers to their committee members. The chairman of the Steering Committee should read the 20 correct answers while the Committees of Four score the papers, allowing 5 credits for each correct answer. The chairmen should find the average score of the committees whose papers were distributed to them. Finally all average scores should be reported to the Steering Committee, who will decide the winning Committee of Four.

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER VI

In Chapters V and VI you have learned among other things how to construct geometric figures and how to measure areas and perimeters. You have discovered by experience that Algebraic formulas are great helps in your work. Thus, you have learned that the work of arithmetic is very much simplified by both geometry and algebra. You will come to understand in Part II that all three work together to make your work easier and to solve the problems of Our Everyday Life and Business.

Before leaving Part I take the End-of-Chapter trial of the three I. R. B. tests of this last chapter. Enter your scores in the proper column.

When you have finished Part I examine the records of your I. R. B. to see how you can improve the Improvement Record Book for Part II. Do not merely follow the directions of the textbook in Chapters VII–XII. Use your own judgment about adding other records. Make your I. R. B. worthy of yourself and of your Junior High School.

PART II

USING THE NEW MATHEMATICS IN OUR EVERYDAY BUSINESS

CHAPTER VII

GETTING READY FOR EVERYDAY BUSINESS. DECIMALS

The Business Man. — When we speak of business men, we usually think of Henry Ford or John D. Rocke-

HENRY FORD

This picture shows Mr. Ford standing between his first car and his ten millionth.

feller, or some rich man who owns a large business. But clerks, lawyers, or merchants are also business men. Even boys and girls have a business, whether it is selling papers, helping with the housework, running errands, or going to school. Our world is a business world.

As you grow up and go into business seriously you will use more and more what you learn in this book. You will estimate and measure as on page 1; you will keep a cash account as Jack Wood did; you will use the four aids and fractions; and you will have geometry help you to construct graphs showing the growth of your business.

Preparing for Business. — In getting ready for business you must learn some of its language. You need to know about percentage, interest, profit and loss, and enough of banking to be able to put your savings in the school bank.

Most business is done with money. Our American money follows what is called the *decimal system*, which is based on the number *ten* and multiples of ten. So, to prepare for business we must get acquainted with this system. Like many other things in the New Mathematics it will make your work quicker and easier. This is especially true of decimal fractions, which make our money system so simple.

Decimals. See the Point! — A decimal is a fraction written like a whole number. As its denominator is always ten, one hundred, one thousand, or the like, this is indicated by a point placed before one or more of the figures. To the left of the point is the whole number (integer), to the right the decimal fraction. Decimals may be changed to common fractions and common fractions to decimals. One half is written .50 and one quarter .25, just as you think of a half dollar as 50 cents or a quarter as 25 cents.

Often, decimal fractions offer a simpler way of doing accurate work than common fractions. Simple fractions like $\frac{1}{3}$, $\frac{1}{8}$, and $\frac{5}{8}$ are usually easier to use than their decimal equivalents $.333^{+}$, .125, and .625, but large fractions like $\frac{5}{125}$ and $\frac{65}{433}$ are clumsy, difficult tools to work with when compared with their decimal equivalents .04 and .15.

You should be very careful to distinguish between whole numbers and decimals, and especially so when reading them. Watch the decimal point.

Illustrative Examples.

.5 is read " five tenths."
.56 is read " 56 hundredths."
.567 is read " 567 thousandths."
.5675 is read " 5675 ten-thousandths."
56.75 is read " 56 and 75 hundredths."
$.03\frac{3}{4}$ is read " three and three-fourths hundredths."

Reading and Writing Decimals

1. How may $\frac{1}{10}$ be written as a decimal? $\frac{5}{100}$? $\frac{18}{1000}$? $\frac{77}{100}$? $\frac{9}{10000}$?

Read each of the following[1]:

2. 6798. **3.** .6. **4.** .78. **5.** .36.

---

[1] Key:
$\frac{1}{10}$ = .1, one tenth
$\frac{1}{100}$ = .01, one hundredth
$\frac{1}{1000}$ = .001, one thousandth
$\frac{1}{10000}$ = .0001, one ten-thousandth
$\frac{1}{100000}$ = .00001, one hundred-thousandth
$\frac{1}{1000000}$ = .000001, one millionth

6. 8.7.
7. 2.08.
8. .375.
9. 2.002.
10. 6.0286.
11. .000036.
12. $.12\frac{1}{2}$.
13. $6.23\frac{1}{3}$.

14. How many cents in .5 of a dollar? In .05 of a dollar? In .50 of a dollar?

Write in decimal form with figures each of the following expressions:

15. Eight and three hundredths; one hundred four ten-thousandths.

16. Ninety-four and six hundredths.

17. Five and one fourth of a tenth.

18. Five and one-fourth tenths.

19. Six thousand six and six hundredths.

20. Three and three hundred forty-five millionths.

Addition and Subtraction of Decimals. — Remember that decimals are like whole numbers except for the decimal point. Watch the position of the decimal point! In addition and subtraction the points must be in a vertical line.

I. R. B. Test [1]. Prepare a chart for Chapter VII (6 tests). Solve the first five examples below. Enter First Trial scores. Your Steering Committee and teacher should determine a standard time for this and the other I. R. B. tests in this chapter.

Decimal Drill

Add and check. See page 9.

1.	**2.**	**3.**	**4.**	**5.**
36.5	85.7	768.092	574.32	452.075
29.45	3.326	26.781	38.42	12.904
3.82	236.478	127.271	7.672	2617.863
11.475	63.082	8.326	218.432	518.206

6. Add: 57.85, 126.38, 67.03, 567.35, 45.654.

7. Find the total sales of each department in the following weekly report, also the daily totals and the grand total.

DEPT.	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTAL
1	$456.75	$549.67	$475.65	$375.46	$380.50	$425.65	
2	258.18	325.55	375.54	278.56	286.78	386.75	
3	125.50	156.76	175.62	148.48	156.78	185.75	
Totals							

COACH AND PLAYER GAME FOR DECIMALS

Let us try the Coach and Player game with these examples. The purpose of the game is to help each pupil to develop all possible skill in mathematics. Let pupils who have made good I. R. B. records act as coaches for those who need to develop more skill.

Divide the class evenly between coaches and players. Let your teacher and Steering Committee decide which coach will work with which player. Change coaches and players occasionally. The coach is a helper only; he should show how a mistake was made and let the player correct it. When the coach finds that he too needs help, the Steering Committee will advise.

Then the player, after the coaching period is finished, may want to work the examples again in out-of-school hours. In athletic games, practice follows coaching. This is the best way for a player to improve his skill.

If your class and your teacher vote favorably upon continuing the Coach and Player game in this chapter, you may wish to change coaches and players from time to time. But remember that all games are optional with your teacher, who knows best when they are appropriate.

8. Subtract 18.25 from 32.6; 23.7 from 61.32.

I. R. B. Test [2]. Enter First Trial scores. Work examples 9–13 for this test.

Subtract each of the following and check:

9.	**10.**	**11.**	**12.**	**13.**
567.875	126.7524	$2567.87	34567.821	$345.67
88.986	97.8635	1578.98	9568.965	257.98

14. Subtract the second column from the first column.

Selling Price	Cost of Goods Sold	Margin (Gross Gain)
$25	$20	
15.09	12.28	
10.50	8.75	
15¢	12¢	
5¢	3¢	

15. Subtract one hundred thirty-six and seventy-five thousandths from four hundred and four thousandths.

Multiplication of Decimals is illustrated by the following:

Illustrative Examples.

1. $.3 \times .5 = \frac{3}{10} \times \frac{5}{10} = \frac{15}{100} = .15.$

2. $.3 \times .05 = \frac{3}{10} \times \frac{5}{100} = \frac{15}{1000} = .015.$

3. $.12 \times .14 = \frac{12}{100} \times \frac{14}{100} = \frac{168}{10000} = .0168.$

4. $2.5 \times .05 = \frac{25}{10} \times \frac{5}{100} = \frac{125}{1000} = .125.$

In each answer above there are as many figures to the right of the decimal point as there are in the multiplier and multiplicand together.

From these we learn the following rule:

Multiply the numbers in the usual way and then from *the right toward the left point off as many decimal places as there are in the multiplicand and multiplier together.*

I. R. B. Test [3]. Enter First Trial scores. Multiply each of the following mentally:

1. .2 × .3.
2. .4 × .2.
3. .5 × .3.
4. .6 × .4.
5. .8 × .5.
6. .12 × .4.
7. .9 × 5.
8. .12 × .6.
9. 1.1 × .4.
10. .05×.07.
11. 2.7×.06.
12. .013×.004.

ORAL AND WRITTEN WORK FOR COACH AND PLAYER CONTEST

In the following exercise of 20 examples let the Steering Committee and teacher decide which examples should be worked mentally. The player of each couple should then give the oral answers to his coach; the coach should write the answers as given and check them. Then the coach should have the player work on paper each example for which an incorrect answer was given. This should be continued until all answers are correct. The first two couples to secure correct answers for all problems which were selected for oral work should choose sides for the written contest; choose couples, not individuals.

In the written contest each coach should work all problems as quickly as possible, so as to be free to assist his player; he should not work the examples for the player.

When all have finished, the Steering Committee should read the correct answers, while coaches and players correct their own answers and count their number of "rights." The written contest will be won by the side having the larger total of rights.

Practice in Products

Find the product of each of the following:

1. 245 × .12. **2.** 563 × .09. **3.** 1275 × .75.
4. 36.5 × 25.8. **5.** .564 × 5.36.

6. What is the cost of 254 bbls. of apples at $4.95 a barrel?

7. Multiply each of the following numbers by 10.
2.5 .62 15.8 1.86 .2 215.06
State a rule for moving the decimal point when multiplying by 10.

8. Multiply each of the following numbers by 100.
4.7 20.5 5.2 2.86 .02 3.046 14.8
What is the rule for multiplying by 100?

9. Multiply each of the following numbers by 1000.
5.136 24.045 .0564 4.34 15.68
What is the rule for multiplying by 1000?

10. Multiply 4.5 by 20; by 30; by 50; by 60; by 70; by 80. Can you state an easy way to multiply by 20? by 30? by 60?

11. Multiply forty-two and fifty-four thousandths by eight and five hundredths.

12. If a farmer sold 156.5 bushels of wheat at $1.05 a bushel, how much should he receive?

13. 20.5 yards of cloth at $1.55 a yard = ?

14. Multiply each of the following and then add the products.

36.8	4.79	18.42	3.86	298.4
.23	.3	2.06	.337	9.66

15. In former days a man bought 15 sheep at $.61 apiece. What did they cost him?

16. In 1 rod there are 16.5 feet. How many feet in 35.75 rods?

17. An automobile made a uniform speed of 45.56 miles an hour for 6.5 hours. How far did it go?

18. Lieut. Commander Richard E. Byrd flew to the North Pole and back in an airplane at an average speed of 85.75 miles an hour for 15.50 hours. How far did he go?

Lieut. Commander Byrd on his Way to the Pole

19. A speedboat made an average speed of 48.25 miles an hour for 3 hours. How far did it go?

20. My automobile tire has a diameter of 34.5 inches. If the distance around it is 3.14 times the diameter, what is the circumference?

21. The Twentieth Century Limited traveled 67.25 miles an hour for 2.16 hours. How far did it go in that time?

22. A cow was tied to a stake by a rope 20.5 feet long. What was the circumference of the circle of grass she could eat in the pasture where she was grazing? See pages 147 and 148.

Division of Decimals. – Decimals may be divided (I) by a whole number, or (II) by another decimal.

(I) When the divisor is an integer the division is performed as shown below.

Illustrative Examples. State the quotient in each of the following:

6 cents ÷ 2 = 3 cents   24 fifths ÷ 6 = $\frac{4}{5}$ .8 ÷ 4 = .2
9 dollars ÷ 3 = 3 dollars 15 tenths ÷ 5 = $\frac{3}{10}$ .21 ÷ 7 = .03

The above examples illustrate the following rule.

If a decimal number is divided by an integer, the quotient has the same number of decimal places as the number of places in the dividend.

Dividing Decimals

I. R. B. Test [4]. Enter First Trial scores. Work examples 1–12 for this test.

1. .18 ÷ 9.
2. .45 ÷ 5.
3. 4.326 ÷ 3.
4. .036 ÷ 6.
5. .425 ÷ 5.
6. 9.808 ÷ 8.
7. 56.16 ÷ 12.
8. 46.54 ÷ 10.
9. 18.92 ÷ 100.
10. .6 ÷ 30.
11. .006 ÷ 600.
12. .0084 ÷ 40.

13. Can you state an easy way of dividing by 10? By 100? By 1000? By 20? State your easy way.

(II) When the divisor is a decimal we divide as follows:

You will observe that 6 ÷ 3 = 60 ÷ 30.

Also 4 ÷ 2 = 40 ÷ 20 or 400 ÷ 200 or 4000 ÷ 2000.

From a study of the above examples you will note that the quotient is the same if we multiply both the dividend and the divisor by 10, 100, or 1000. Now let us work the following

Illustrative Example 1. .024 ÷ .6.

Solution: If we can get rid of the decimal in the divisor, we shall have a simple example like those under (I) on the preceding page. So we multiply both the dividend and the divisor by 10. We now have .24 ÷ 6. This gives us an example like those in (I) above and our result is .04.

Illustrative Example 2. Divide 4.864 by .16.

Solution:

$$\begin{array}{r} 30.4 \\ {}_{\times}16.\overline{)4_{\times}86.4} \end{array}$$

Explanation. To get rid of the decimal in the divisor the divisor and dividend are each multiplied by 100, thus moving the decimal point to the right of the 6 in the divisor and making it an integer. The point is moved two places to the right in the dividend. The old decimal points are crossed out.

The division is then performed as in the division by an integer. The quotient has as many decimal places as the number of places that remain in the new dividend.

The above examples lead us to the following rule:

To divide by a decimal

1. *Move the decimal point of the divisor to the right end of the divisor.* 2. *Make a new dividend by moving the decimal point to the right as many places as the decimal point of the divisor has been moved.*

3. *Divide as in whole numbers.* 4. *Point off as many places in the quotient as remain in the new dividend.*

Dividing Decimals by Decimals

Find the quotient in each of the following:

1. .456 ÷ .03.
2. 45.6 ÷ .03.
3. 4.56 ÷ .03.
4. 456 ÷ .03.
5. 456 ÷ .003.
6. .875 ÷ 2.5.
7. 6.25 ÷ 2.5.
8. 16.9 ÷ 1.3.
9. 229.46 ÷ .0004.

10. A tank contains 1860 cubic inches. A gallon of water contains 231 cubic inches. How many gallons will the tank hold?

11. A bushel contains 2150.42 cu. in. How many cubic inches in a peck? in a quart?

12. A farm containing 234.85 acres was divided into lots of .35 of an acre each. If 3.5 acres were used up for streets, how many lots were there?

13. Sheets of paper .02 of an inch thick were piled up into a pile which was 3.8 inches thick. How many sheets in the pile?

14. A railroad company laid 3745 miles of track at a cost of $17,827,885.25. What was the average cost a mile?

15. The product is .002814, the multiplier is .067. Find the multiplicand.

Changing Decimals to Common Fractions. —

Illustrative Example 1. Change .25 to a common fraction.

Solution: $.25 = \frac{25}{100} = \frac{1}{4}$.

What do you do to $\frac{25}{100}$ to obtain $\frac{1}{4}$?

Example 2. Reduce $.12\frac{1}{2}$ to a common fraction.

Solution: $$.12\tfrac{1}{2} = \frac{12\frac{1}{2}}{100} = \frac{25}{2} \div 100 = \frac{\overset{1}{\cancel{25}}}{2} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{1}{8}.$$

Example 3. Change the mixed decimal 4.75 to a mixed number.

Solution: $4.75 = 4\frac{75}{100} = 4\frac{3}{4}$.

To change decimal fractions to common fractions, write the numerator over the denominator and reduce the result to the lowest terms.

Reducing Decimals to Common Fractions

I. R. B. Test [5]. Enter First Trial scores. Use examples 1–20 for this test.

Reduce each of the following to a common fraction:

1. .5.
2. .75.
3. .45.
4. .20.
5. .50.
6. .35.
7. .12.
8. .24.
9. .375.
10. .875.
11. .1250.
12. .0005.
13. $.37\frac{1}{2}$.
14. $.62\frac{1}{2}$.
15. $.87\frac{1}{2}$.

Reduce the following to mixed numbers:

16. 12.5. **17.** 25.25. **18.** 5.75. **19.** 16.125. **20.** 100.500.

Changing Common Fractions to Decimals is illustrated by the following examples.

Illustrative Example 1. Reduce $\frac{3}{5}$ to a decimal fraction.

Solutions: (*a*) $\frac{3}{5} \times \frac{2}{2} = \frac{6}{10} = .6$ or (*b*)

$$\begin{array}{r} .6 \\ 5\overline{)3.0} \\ \underline{3\,0} \end{array}$$

Can you tell why we multiplied numerator and denominator by 2 in (*a*)? How did we obtain the 2?

Example 2. Change $\frac{4}{25}$ to a decimal.

Solutions: (*a*) $\frac{4}{25} \times \frac{4}{4} = \frac{16}{100} = .16$

or (*b*)

$$\begin{array}{r} .16 \\ 25\overline{)4.00} \\ \underline{2\,5} \\ 150 \\ \underline{150} \end{array}$$

Notice that in (*b*) we keep adding ciphers to the numerator as long as we desire to do so.

Example 3. Reduce $\frac{2}{3}$ to thousandths.

Solution:

$$\begin{array}{r} .666\frac{2}{3} \\ 3\overline{)2.000\phantom{\frac{2}{3}}} \end{array}$$

Changing Fractions to Decimals

I. R. B. Test [6]. Enter First Trial scores. Use examples 1–20 for this test.

Change each of the following to decimals:

1. $\frac{1}{4}$.
2. $\frac{1}{2}$.
3. $\frac{1}{5}$.
4. $\frac{1}{3}$.
5. $\frac{1}{12}$.
6. $\frac{3}{8}$.
7. $\frac{5}{8}$.
8. $\frac{7}{8}$.
9. $\frac{7}{20}$.
10. $\frac{3}{200}$.
11. $\frac{2}{50}$.
12. $\frac{3}{16}$.
13. $\frac{9}{64}$.
14. $\frac{3}{75}$.
15. $\frac{18}{40}$.
16. $\frac{24}{5}$.
17. $\frac{36}{8}$.
18. $\frac{425}{16}$.
19. $\frac{7}{12}$.
20. $\frac{40}{16\frac{2}{3}}$.

Obtaining Results to a Desired Accuracy. — You may often be called upon to obtain decimal results correct to the nearest tenth or the nearest hundredth, etc.

Illustrative Example 1. $\frac{19}{25}$ gives us what value decimally to the nearest tenth?

Solution: $\frac{19}{25} \times \frac{4}{4} = \frac{76}{100} = .76$. The nearest tenth is $.8^-$.

Example 2. I bought a tire for $38 and it ran 20,000 miles. What did it cost me per mile to the nearest mill?

Solution: $38 ÷ 20,000 = $.0019, or to the nearest mill it cost 2 mills per mile. What would it have cost per mile to the nearest mill had it run only 8000 miles?

Finding Quotients to the Nearest Tenth

Divide the following, correct to the nearest tenth.

1. 5.59 ÷ 2.
2. 63.18 ÷ 3.
3. 74.28 ÷ 5.
4. 7.19 ÷ 12.
5. 125.29 ÷ 17.
6. 89999.52 ÷ 57,692.
7. 106.35 ÷ 25.
8. 472.50 ÷ 375.
9. 80.64 ÷ 896.

REVIEW TRIAL OF I. R. B. TESTS, CHAPTER VI

Try again the three I. R. B. tests of Chapter VI. On the chart record your Review Trial scores in the proper column. Look over the directions given in the Review Trial of Chapter I, page 40.

Baseball Standings

In the baseball leagues the standings of the different teams are obtained by dividing the number of games won by the total number of games played. The result is carried out to the nearest thousandth. In the following tabulation *W* stands for games won and *L* stands for games lost during the season. Find to the nearest thousandth the standings of the teams.

The Pittsburgh Pirates Warming Up

NATIONAL LEAGUE

	W	L	Standing
Pittsburgh	95	58	
New York	86	66	
Cincinnati	80	73	
St. Louis	77	76	
Boston	70	83	
Brooklyn	68	85	
Philadelphia	68	85	
Chicago	68	86	

AMERICAN LEAGUE

	W	L	Standing
Washington	96	55	
Philadelphia	86	66	
St. Louis	82	71	
Detroit	81	73	
Chicago	79	75	
Cleveland	70	84	
New York	69	85	
Boston	47	105	

Averages in Whole Numbers and in Decimals. — Business men often have to figure averages. This is done by dividing the sum of several like quantities by the number of quantities, as we saw in Chapter I. When an average contains a fraction this is usually changed into a decimal.

Example. In the same store the average sales per day during the first week of October was $67; second week $63; third week $65; and the fourth week $58. What were the average sales per day during the four weeks of October?

Solution: $67 + $63 + $65 + $58 = $253.
$253 ÷ 4 = $63.25

Averages

1. The weights of six boys were 85, 80, 95, 75, 82, and 87 pounds. What was the average weight per boy?

2. A family's expenses this year were $228 more than they were last year. What was the average monthly increase?

3. If a man saves $754 in a year, what is his average weekly saving?

4. An automobile traveled 81 miles in 2.5 hours. What was its average speed in miles per hour?

5. A boy spent 15¢, 20¢, 18¢, 19¢, 17¢, and 19¢ during 6 weeks. Find his average spendings per week.

6. Mabel Johnson's father paid the following electric light bills for nine months: $1, $1.77, $2.06, $2.21, $1.68, $1.68, $1.54, $1.24, and $.95. What was the average cost per month?

7. A certain chain store organization sold $239,020,368 worth of goods in 1925. What were its average sales per month?

8. If that same organization had 1415 stores, what were their average sales per store per month?

9. If the cost of running the city of New York is $402,000,000 in one year and there are approximately 6,000,000 people in that city, what is the average cost per person? (Called per capita cost.)

10. Find out the total cost of running your city or town for one year and then find out the number of people in your city or town as given by the last census or by your teacher. Then find the average cost per person.

11. The cost of a 50-trip ticket from a large city to one of its suburbs is $9.90. What is the average cost per trip?

12. Six clerks in a store sold the following amounts one day: $75.68, $50.65, $82.36, $68.46, $90.12, and $62.14. What were the average sales per clerk? How many were above the average? below?

13. If the running time of the 20th Century Limited from

THE TWENTIETH CENTURY LIMITED

New York to Buffalo is 9 hours and the distance is 440 miles and the train should make 4 stops of five minutes each, what would be the average running speed per hour of the train?

14. Mr. Johnson made a five-day automobile trip through California and Oregon, covering the following distances each day: 300 miles, 280 miles, 260 miles, 290 miles, and 295 miles. What was the average distance traveled per day?

15. A farmer sold six cows for $120, $80, $90, $115, $75, and $96 respectively. What was the average price per head?

16. Mr. Ostrander made an automobile trip of 1017.5 miles and used 55 gallons of gasoline. What was his average mileage in miles per gallon of gasoline?

17. If coal costs $13.50 a ton and a family used on the average $1\frac{1}{2}$ tons a month for 8 months, how much would their coal cost?

18. The 1925 World Almanac gives the population of the ten largest cities of the country in 1920 as follows:

New York . . .	5,839,746	St. Louis . . .	788,375
Chicago . . .	2,833,288	Boston	764,549
Philadelphia . .	1,894,500	Baltimore . . .	762,222
Detroit	993,678	Los Angeles . .	622,343
Cleveland . . .	854,565	Pittsburgh . .	607,902

What is the average population of these ten cities? What is the shortest way to divide by ten?

19. Find the average number of pounds of butter fat per cow from the following:

TOTAL NUMBER OF POUNDS OF BUTTER FAT	NUMBER OF COWS	AVERAGE PER COW
348.8	8	
455.3	11	
880.	25	

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER VII

In this chapter you have practiced the interchange of common fractions and decimals; you have used the fundamental operations with decimals, you have observed carefully the proper placing of the decimal point, and you have learned that decimals are tools of everyday use.

Now give yourself, outside of school hours, a test of your skill with decimals. For this purpose try the six I. R. B. tests of this chapter. Enter your new scores in the End-of-Chapter column. Remember that you alone must determine your 100% trustworthiness score.

Cumulative Review

1. What will it cost to build a cement walk 4 feet wide and 32 feet long at $1.40 a square yard?

2. If the divisor is 54.62, the quotient 74.86, and the remainder is .1874, what is the dividend?

3. One side of a square field is 40 rods. How many acres in the field?

4. A large ship made the following record for five hours: 22.6 mi., 23.4 mi., 25.3 mi., 24.9 mi. and 23.8 mi. What was the average speed per hour?

5. An automobile cost $1085. If it depreciated (lessened in value) $\frac{3}{10}$ of its cost the first year, $\frac{1}{5}$ of its cost the second year, and $\frac{1}{10}$ of its cost the third year, how much was it worth at the end of the third year?

6. Arrange each of the following in order of size, the largest first: .6, $\frac{1}{2}$, 2.7, $\frac{3}{4}$, 12.75, $9\frac{3}{4}$, .9, $11\frac{1}{2}$, .4, 7.5, and $4\frac{3}{4}$.

Hint: Change each to a decimal or mixed decimal.

7. The sides of a triangle are 4 feet 7 inches, 3 feet 3 inches, and 4 feet 8 inches. What is the perimeter of the triangle?

8. The sales of a company for 1926 were $136,752,840, and if this was \$9,946,327 more than in 1925, what were their sales in 1925?

9. A triangular city lot has a base of 62 feet and an altitude or depth of 150 feet. What will it cost at 90 cents a square foot?

10. The distance around a circular tank is 125 feet. Find the radius of the base. (Use $\pi = 3.1416$.)

What is the area of the bottom of the tank?

Practice With Formulas

Each formula given at the right fits some statement given at the left. Mark with corresponding numbers the formulas that belong to the numbered statements at the left.

1. The area of a circle equals pi times the radius squared.

2. The area of a triangle equals one half of the base times the altitude.

3. The area of a square is the square of a side.

4. The area of a trapezoid is one half the altitude times the sum of the bases.

5. The perimeter of a rectangle is twice the sum of two adjacent sides.

6. The area of a rectangle is the product of the base and the altitude.

7. The area of a parallelogram is the base times the altitude.

8. The radius of a circle equals the circumference divided by twice pi.

9. The circumference of a circle is two times pi times the radius.

$A = bh.$

$C = 2\pi r.$

$A = \pi r^2.$

$A = \frac{1}{2} ba.$

$A = s^2.$

$A = \frac{1}{2} a(b + b').$

$P = 2(a + b).$

$A = ab.$

$r = \frac{C}{2\pi}.$

Practice in Finding Squares

1. $2^2 =$
2. $3^2 =$
3. $4^2 =$
4. $9^2 =$
5. $8^2 =$
6. $7^2 =$
7. $6^2 =$
8. $5^2 =$
9. $10^2 =$
10. $12^2 =$
11. $11^2 =$
12. $15^2 =$
13. $20^2 =$
14. $25^2 =$
15. $50^2 =$
16. $.5^2 =$
17. $.9^2 =$
18. $1.2^2 =$
19. $.07^2 =$
20. $2.5^2 =$

CHAPTER VIII

THE FOUNDATION OF EVERYDAY BUSINESS. PERCENTAGE

Per Cent. — Every business man talks in terms of per cent. He buys on a per cent discount. He sells on a per cent gain. He estimates the per cent cost of rent, wages, fuel, light, taxes, etc. He gives a per cent increase to his employees. He puts a per cent of his profits into enlarging his business.

Thus the business man talks, thinks, and actually lives in per cents. He will tell you that you should learn how to use per cent. He wants only 100 per cent work. Do 100 per cent work in your mathematics and you may be sure that the 100 per cent business man will have a place for 100 per cent assistants.

JUDGE ELBERT GARY

Judge Gary was president of the United States Steel Corporation and one of America's biggest business men.

Per cent comes from the Latin *per centum*, meaning *by the hundred*. So in per cent we figure by hundredths.

Per cents are simply decimals like those we have just been studying, but they are easier because they are always hundredths.

Suppose we change each of the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{5}{4}$ first to a fraction having 100 as a denominator, and then to a decimal. We have

$$\frac{1}{2} = \frac{50}{100} = .50 \qquad \frac{1}{5} = \frac{20}{100} = .20$$
$$\frac{1}{4} = \frac{25}{100} = .25 \qquad \frac{2}{5} = \frac{40}{100} = .40$$
$$\frac{3}{4} = \frac{75}{100} = .75 \qquad \frac{5}{4} = \frac{125}{100} = 1.25$$

Since 50% is only another way of saying 50 hundredths, the decimal .50 is called " 50 per cent." The symbol for per cent is %, so we write 50 per cent as 50%. This sign is shorthand for the words " per cent." In the same way the other fractions of the above may be written 25%, 75%, 20%, 40%, and 125%.

Likewise: $\quad 1 \text{ unit} = \frac{100}{100} = 100\%.$

$$\frac{1}{3} = \frac{33\frac{1}{3}}{100} = .33\tfrac{1}{3} = 33\tfrac{1}{3}\%. \qquad \frac{2}{3} = \frac{66\frac{2}{3}}{100} = .66\tfrac{2}{3} = 66\tfrac{2}{3}\%.$$

$$\frac{1}{6} = \frac{16\frac{2}{3}}{100} = .16\tfrac{2}{3} = 16\tfrac{2}{3}\%. \qquad \frac{1}{8} = \frac{12\frac{1}{2}}{100} = .12\tfrac{1}{2} = 12\tfrac{1}{2}\%.$$

$$\frac{1}{200} = \frac{\frac{1}{2}}{100} = .00\tfrac{1}{2} = \tfrac{1}{2}\% \ (\tfrac{1}{2} \text{ of } 1\%).$$

Fractions, decimals, and per cents are three different tools to do the same work in mathematics. Sometimes one tool is easier to use in your work and sometimes another. You should learn how to use each tool in order to choose the best tool for each piece of work. Be sure that tools of the same size are exchanged for each other, *e.g.*, $\frac{1}{4}$, .25, 25%. Remember that an even exchange never makes trouble.

C. C. AND C. C. GAME

The following 50 examples will make a splendid exercise for the C. C. and C. C. Game (see chapter II). In this case have your Steering Committee act as the Coaching Committee who, with your teacher's advice, will divide the class into Coaching Couples; based on former I. R. B. records there should be one high score coach and one low score pupil in each couple.

Let the pupil of each couple work the examples while the coach dictates them and checks answers. The coach should repeat each example until the pupil secures the correct answer. In case of doubt call upon the Coaching Committee.

See which Coaching Couple can finish the 50 examples in the shortest time. If any couple wishes to improve the time record, try the exercise again in out-of-class hours at school or at one of the homes of the couple. Perhaps your teacher will indicate which couples can improve their skill by this additional practice.

An Even Exchange

Fractions = Decimals = Per Cent

State or write each of the following fractions as a decimal and then as a per cent.

1. $\frac{3}{8}$.
2. $\frac{5}{8}$.
3. $\frac{7}{8}$.
4. $\frac{3}{5}$.
5. $\frac{4}{5}$.
6. $\frac{1}{6}$.
7. $\frac{5}{6}$.
8. $\frac{3}{2}$.
9. $\frac{6}{5}$.
10. $\frac{7}{4}$.

Write each of the following as a decimal:

11. 3%.
12. 9%.
13. 15%.
14. 28%.
15. 36%.
16. 45%.
17. 56%.
18. 67%.
19. 80%.
20. 125%.
21. 150%.
22. 275%.
23. $\frac{1}{4}$%.
24. $\frac{1}{2}$%.
25. $1\frac{1}{2}$%.
26. $37\frac{1}{2}$%.

Express with the % sign.

27. .04.
28. .13.
29. .30.
30. $.04\frac{1}{2}$.
31. $.16\frac{1}{4}$.
32. .285.
33. $.75\frac{1}{2}$.
34. $.07\frac{1}{2}$.
35. .125.
36. 1.50.
37. 2.25.
38. 5.75.

39. 20% of a number is how many hundredths of it?

40. A baseball team won 8 out of the 10 games that it played. What part of all games did it win? What per cent of all?

41. What common fraction is represented by

25%?
50%?
75%?
$12\frac{1}{2}$%?
15%?
60%?
$37\frac{1}{2}$%?
$62\frac{1}{2}$%?
80%?
125%?
150%?
275%?

42. Five-eighths of a number is what per cent of it?

THE WRECKED AUTOMOBILE

43. An automobile was wrecked after it had gone 75 miles of a 225-mile trip. What part of the trip did it make? What per cent?

44. John was present only 15 days of a month of 20 school days. What part of the month was he present? What per cent? What per cent was he absent?

45. It cost \$500 to sell a \$10,000 house. What part of the selling price did it cost to sell? What per cent?

46. A salesman earns \$1800 in a year. What part of his yearly salary does he save if he spends \$1500? What per cent?

47. It is 9 miles from town A to town B. James Marsden rode 6 miles and walked the rest of the distance. What part did he walk? What per cent of the total distance?

48. In a school of 4000 pupils, 80 pupils were absent. What part were present? What per cent?

49. 5 days of a certain 30-day month were rainy. What part were not rainy? What per cent?

50. 175% of a number is how many hundredths of it?

The First Use of a New Tool. Finding the Per Cent of a Number. — Mary's teacher told her that 12% of her class were doing "A" work. The class consisted of 25 pupils. Mary was anxious to find out how many pupils were being marked A. How could she do it?

In our work with fractions we saw that $\frac{1}{2}$ *of* meant $\frac{1}{2}$ *times*.

For example: $\frac{1}{2}$ of 10 apples means $\frac{1}{2} \times$ 10 apples or 5 apples.
Similarly $\frac{1}{3}$ of 12 bushels $= \frac{1}{3} \times$ 12 bushels $=$ 4 bushels.
And $\frac{1}{5}$ of 15 dollars $= \frac{1}{5} \times$ 15 dollars $=$ 3 dollars.

In the same way, 12% of the class means .12 times the number in the class or $.12 \times 25$ or 3 pupils. Remember about the decimal point in multiplying decimals.

Starting the New Tool to Work

Do-It-In-Your-Head As-Far-As-You-Can

Find

1. 5% of $200.

2. 4% of 825 bu.

3. 6% of 240 quarts.

4. 28% of 1500 feet.

5. 20% or ⅕ of 625 pupils.

6. 60% of $1800.

7. 70% of 5000 miles.

8. 12½% of 560 boys.

9. 18% of 6000 pencils.

10. 10% of 45 yards.

11. 200% of $600.

12. 350% of $100.

13. 56% of 80.

14. 2½% of 200.

15. Find 1% of each of the following: 100, 400, 850, 1200, $1575. Can you state an easy way to find 1% of a number? State it.

16. Having discovered this easy way of finding 1% of a number, how then could you find 4% of the number? 3% of it? 7% of it? 12% of it?

17. Find 8% of each of the following: 1500, $20,000, 5000 bu., 6750.

18. Find ½% (½ of 1%) of each of the following: 2000, $4000, $5764.

19. Find ¼% of each of the following: $400, $1200, $4264, $234.04.

20. 306% of 800 miles = ?

21. A concrete road 12½ miles long and running over a mountain cost approximately $900,000. If the contractor received 10% for his work in superintending the job, how much would he receive?

22. Auctioneers receive about 5% of the selling price of the goods that they sell. If the sales at an auction amounted to $2560, how much would the auctioneer receive? How much would the owner of the goods receive?

23. An architect was paid 5% of the cost of a house for drawing the plans and supervising the building of it. If the house cost $11,760, how much did he receive as his fee?

24. Some realtors charge 3% of the value of a house for selling it. My house was sold for $8250. How much did the realtor receive for making the sale?

25. In a high school which had 450 boys, 8% of them tried for the baseball team. How many tried?

Finding Percentage by Algebraic Formula. — We usually call per cent *rate per cent*, or simply *rate*, or *fractional part*. The number of which we find the per cent is called the *base* or *whole*. The result of our work is called *percentage* or *part*. We must get clearly in mind the difference between per cent and percentage. Per cent is the rate; percentage is the result or part.

Illustrative Example. Find 8% of $160.

Solution: $8\% \text{ of } \$160 = \underset{\text{rate}}{.08} \times \underset{\text{base}}{\$160} = \underset{\text{percentage}}{\$12.80}.$

We have, therefore, the following rule:

To find the percentage, multiply the base by the rate.

Or more simply *percentage* = *base* × *rate.*

If we use only the initial letters of the words, we have $p = b \times r$, or written algebraically $\mathbf{p = br}$.

This is the formula for percentage.

Algebra Helps Arithmetic Do Your Work

In the following examples use the above formula to find the percentage.

1. Base $500, rate = 6%.

2. Base 600, rate = 35%.

3. Base 800 bu., rate = 40%.

4. B, 1260, r = 75%.

5. B, $1236.75, r = 25%.

6. B, $1500, r = 67%.

7. B, 5675, r = 12%.

8. B, 9680, r = 46%.

9. B, 126.84, r = 15%.

10. B, 25.74, r = 26%.

Everyday Problems in Percentage

Estimate your answers, writing the estimates in a vertical column. When you have completed your estimates, work the problems on paper. Compare your answers with your estimates by writing the answers in a second vertical column. In cases where estimates and answers differ to a large extent, check by working the problem again. What proportion of sensible estimates can you make?

1. A man receiving a yearly salary of $3600 saved 20% of it. How much did he save?

2. John had 60 cents in the morning. He spent 15% of it during the day. How much did he spend? How much had he left?

3. Find the value of 36% of 2000 bushels of wheat at $1.15 a bushel.

4. What is the value of 75% of 256 acres of land at $56 an acre?

5. The depreciation (loss of value) of a new automobile is computed at about 30% of its original value for the first year. If an automobile costs $1400 when new, what is its value at the end of the first year?

6. If the same automobile loses 18% of its cost during the second year, what is it worth at the end of the second year?

7. If the automobile cost the owner 4% of its cost to insure it against fire, theft, liability, property damage, and collision, how much insurance cost would he have to pay the first year?

8. Mr. Jones is worth 48% of $50,000 and Mr. Noble is worth 56% of $40,000. Which is worth the more and how much?

9. A suit of clothes was marked $40. During a sale the merchant reduced it 15%. How much did he reduce it? What was the new sale price?

10. If I borrow $3000 to help pay for a house and am charged 6% a year for the use of the money, how much must I pay annually for the use of it?

11. There were 3600 people in attendance at a county fair one day. The next day on account of rain the attendance was 42% less. How many attended the second day?

THE CROWD AT THE COUNTY FAIR

12. Mr. Howard, whose income is $2500, spends 20% of it for rent, 25% of it for food, 15% for fuel and light, 15% for clothing, 10% for car fare, and the balance for savings. How much does he save?

13. Oakland, California, had about 216,300 inhabitants in 1920. If by 1925 the population had increased 12%, how much had it increased in the five years? What would be the new population?

14. Mary was working for \$16 a week but on account of poor business her employer told her he would have to reduce her salary $12\frac{1}{2}\%$. How much was she reduced per week? What was her new salary?

15. A clerk was paid 3% of the amount of the goods that he sold. On a certain day he sold \$340.50 worth of goods. How much did he earn that day?

More Even Exchange. — Business men often change per cents to common fractions by writing 100 as the denominator and reducing to lowest terms.

Illustrative Example 1. What common fraction is equivalent to 5%?

Solution: In order to solve this we write

$$5\% = .05 = \tfrac{5}{100} = \tfrac{1}{20}.$$

Illustrative Example 2. Change $33\frac{1}{3}\%$ to a common fraction.

Solution: $33\frac{1}{3}\% = .33\frac{1}{3} = \dfrac{33\frac{1}{3}}{100} = \dfrac{100}{300} = \dfrac{1}{3}.$

What did we do to both the numerator and the denominator when we changed from $\dfrac{33\frac{1}{3}}{100}$ to $\dfrac{100}{300}$?

Illustrative Example 3. What is the equivalent common fraction for $\frac{7}{8}\%$?

Solution: $\dfrac{7}{8}\% = \dfrac{\frac{7}{8}}{100} = \dfrac{7}{8} \div 100 = \dfrac{7}{8} \times \dfrac{1}{100} = \dfrac{7}{800}.$

Illustrative Example 4. Change 450% to an equivalent mixed number.

Solution: $450\% = \tfrac{450}{100} = 4.5 = 4\tfrac{1}{2}.$

Keeping the Change Correct

I. R. B. Test [1]. Prepare in I. R. B. a chart for the First, End-of-Chapter, Review, and Fourth Trial records (seven tests). Record your first score in the proper column. Have the Steering Committee determine the standard time. Keep your time and accuracy score. Change each of the following per cents to fractions in their lowest terms or to mixed numbers. See if you can do all of them mentally.

1. 20%.
2. 25%.
3. 4%.
4. 6%.
5. 30%.
6. 45%.
7. 80%.
8. 95%.
9. 75%.
10. $37\frac{1}{2}\%$.
11. $66\frac{2}{3}\%$.
12. 65%.
13. $87\frac{1}{2}\%$.
14. $16\frac{2}{3}\%$.
15. 24%.
16. $83\frac{1}{3}\%$.
17. $6\frac{2}{3}\%$.
18. $11\frac{1}{9}\%$.
19. 250%.
20. 400%.
21. 125%.
22. 375%.
23. $\frac{1}{2}\%$.
24. $\frac{1}{4}\%$.
25. $1\frac{1}{2}\%$.

Table of Equivalents. "Have the Exact Change Ready." — Most business men know the per cent equivalents for the most commonly used fractions. The following table contains some important fractions expressed as per cents. You should memorize them. To help your memory, copy the fractions in your I. R. B., and then write the equivalents without referring to the book. Review Aliquot Parts on page 36.

Table of Per Cents

$\frac{1}{2} = 50\%$
$\frac{1}{3} = 33\frac{1}{3}\%$
$\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{1}{4} = 25\%$
$\frac{3}{4} = 75\%$
$\frac{1}{5} = 20\%$
$\frac{2}{5} = 40\%$
$\frac{3}{5} = 60\%$
$\frac{1}{6} = 16\frac{2}{3}\%$
$\frac{5}{6} = 83\frac{1}{3}\%$
$\frac{1}{8} = 12\frac{1}{2}\%$
$\frac{3}{8} = 37\frac{1}{2}\%$
$\frac{5}{8} = 62\frac{1}{2}\%$
$\frac{7}{8} = 87\frac{1}{2}\%$
$\frac{1}{10} = 10\%$
$\frac{1}{12} = 8\frac{1}{3}\%$
$\frac{1}{16} = 6\frac{1}{4}\%$
$\frac{1}{15} = 6\frac{2}{3}\%$
$\frac{1}{20} = 5\%$
$\frac{1}{25} = 4\%$
$\frac{1}{200} = \frac{1}{2}\%$
$\frac{1}{400} = \frac{1}{4}\%$
$\frac{1}{300} = \frac{1}{3}\%$
$\frac{3}{400} = \frac{3}{4}\%$
$\frac{500}{100} = 500\%$

Easy Work with the Right Tool

I. R. B. Test [2]. Have your Steering Committee determine a Standard Time. Do your work mentally. Record your time and accuracy scores in your I. R. B. under First Trial Scores.

Find by the use of the fractional equivalents:

1. 25% of 40.
2. 33⅓% of 63.
3. 75% of 60.
4. 50% of 6.8.
5. 87½% of 24.
6. 87½% of 2.4.
7. 16⅔% of 48 cents.
8. 60% of 5 tenths.
9. 37½% of 32 quarts.
10. 83⅓% of 54 dollars.
11. 12½% of $72.16.
12. 5% of 200.
13. 4% of 75 bushels.
14. ½% of 5000 dollars.
15. ⅕% of 2500 nickels.
16. 62½% of 72 books.

Finding Amount of Increase. — In business many increases in salary are on a per cent basis.

Illustrative Example. Henry, who had been receiving $15 a week, was told by his employer on the first of January that on account of his honest and painstaking work he would have his wages advanced 20% for the coming year. How much would Henry then receive?

Solution:

20% or ⅕ of $15 = $3, increase.
$15 + $3 = $18, new wage. Or

$$\begin{array}{r} 5)\$15 \\ 3 \\ \hline \$18 \end{array}$$

Oral Drill

1. $18 increased by 33⅓% = ?
2. $25 increased by 20% equals what?
3. What is $50 increased by 10% of itself?

4. What is the new price of a suit of clothes now costing \$40 if the price is increased $12\frac{1}{2}\%$ due to increased cost of materials and labor?

The New Road Around the Mountain

5. The distance between two towns over a mountain was 16 miles by the old road. A new improved road around the mountain is 75% longer. How far is it by the new road?

6. How much is 480 increased by 5%?

Written Practice in Finding Amount of Increase

1. The monthly commutation fare from a certain place in Missouri to St. Louis was \$20. If this is increased 20%, what is the new fare?

2. Coal was selling for \$9 a ton, but has increased by $66\frac{2}{3}\%$. What is the new selling price? What is the ratio of the new price to the old? Draw a bar graph illustrating the ratio between both prices.

3. John has been receiving $16 a week, but on account of faithful and loyal work his pay has increased 25%. What is his new wage?

4. Mr. Isbell bought a house for $12,000, but it has increased in value $16\frac{2}{3}\%$. What is its new value?

5. Johnson Bros. had 20,000 square feet of floor space. They built on an addition which increased their floor space by 20%. What was the increase? What is their new floor space?

6. The population of Cleveland, Ohio, in 1920 was 796,841. What will be the population after it has increased 7%?

7. How much is $456 increased by 18% of itself?

8. A farm is now worth $8000. A new improved road by the place was calculated to increase the value of all property about 12%. What should be the new value of this farm?

9. The cost of living was said to be increased by about 90% in 1920 over that of 1913. If it cost a certain family $1200 a year in 1913, what would it have cost them in 1920?

10. How much is 30 increased by $\frac{1}{3}$? 75 increased by $66\frac{2}{3}\%$? 240 increased by 150%? 640 increased by $12\frac{1}{2}\%$? $1218 increased by $83\frac{1}{3}\%$? $15,000 increased by $\frac{1}{5}\%$? 18,000 increased by $\frac{1}{3}\%$? What is the ratio of the increase to the original in each of the above?

Finding Amount of Decrease. — A falling off in business or a cut in salary is often figured on a per cent basis.

Illustrative Example. Mary was told one day that on account of poor business her employer must decrease her present wages of $18 a week by $16\frac{2}{3}\%$. How much would she then receive?

Solution:

$16\frac{2}{3}\%$ or $\frac{1}{6}$ of \$18 = \$3, decrease.
\$18 − \$3 = \$15, new wage.

Or 6)\$18 old wage
3 decrease
\$15 new wage

Oral Drill

1. How much is \$36 decreased by $\frac{1}{12}$?

2. \$75 decreased by $33\frac{1}{3}\%$ equals what?

3. 600 decreased by 7% = ?

4. 7200 miles decreased by $12\frac{1}{2}\%$ is how many miles?

5. A tax of \$40 was decreased by 10% the following year. What was the tax the following year?

Written Practice in Finding Amount of Decrease

1. Henry's present wage of \$16 a week was decreased by 25%. What was his new wage? What was the ratio of the new wage to the old? Illustrate the two wages graphically.

2. The distance between two places on a certain railroad was 24 miles. The company at a great expense built a tunnel through a mountain and decreased this distance by $33\frac{1}{3}\%$. What was the new distance between those places?

3. A certain automobile cost \$1500 when new. If it decreased in value 37% during the first year, what was its value at the end of the first year?

4. A factory in a certain town was valued at \$16,000 in 1924, but because many industries moved away that year, all property was said to decrease in value 40%. What was the new value of that factory?

5. A house and farm were valued at \$3500. A new improved road was built in such a way that it cut across the front lawn of that place about 20 feet from the house. This was claimed to decrease the value of the place by 25%. What was the new value of the property?

6. A man sold an automobile for $560. If the cost to him was 12½% less than he sold it for, what did it cost him?

7. On account of moving the railroad shops out of a certain town the population of 11,580 was decreased by 5%. What was the new population of that town?

8. A man was paying $120 a month rent for a store, but moved into another store which he obtained for 20% less. What was his new rent?

9. A suit was selling for $50, but because of poor business all suits were reduced 15%. What was the new selling price?

10. An office building costing $25,000 was said to lessen in value 3% of the cost each year. What would be its value at the end of 4 years?

The Review System

1. John's salary the first year was $1250. His salary increased 8% during the second year and 12% during the third year, and then on account of poor financial conditions it decreased 10% for the fourth year. What was his salary the fourth year? Draw bar graphs to illustrate the salaries of the four years.

2. What is the complement of 50°? of 67°? of 30° 30′? of 84° 20′?

3. The divisor is .45, the quotient is 21.4. What is the dividend?

4. A man buys eggs at the rate of 2 for 5 cents. He sells them at the rate of 3 for 9 cents. How much will he make on 30 dozen?

5. A train running 64 kilometers an hour is making how many miles per hour?

6. A line is 8 meters 6 centimeters long. How many feet long is it?

7. How much less is $62\frac{1}{2}\%$ of \$7840 than $83\frac{1}{3}\%$ of \$8412?

Finding Rate Per Cent When the Base and the Percentage Are Given. — Helen's father, who was a physician, asked her what per cent of the pupils in her class had been vaccinated. Helen knew that 23 of the 25 members of her class had been vaccinated but she did not know how to change this to per cent. Her father asked her what fraction of the class was vaccinated. She replied "$\frac{23}{25}$." Her father then asked her to change this fraction to a decimal. This she did and obtained .92. Then Helen knew that 92% of the class had been vaccinated.

Then her father gave her the first four of the following examples all of which she worked in 6 minutes. See if you can do better than that. If you have learned your table of equivalents, it will save you much time and work.

Finding the Rate

1. What per cent of 100 is 50? What per cent of 80 is 20? of 60 is 40? of 75 is 25? of 60 is 20? of 25 is 20? of 16 is 12?

2. Mr. Warren put 120 bushels of potatoes in his cellar, but 30 bushels decayed. What per cent were lost?

3. In a mathematics class in the Holmes Junior High school 27 out of 30 passed the examination. What per cent failed? What per cent passed?

4. A lady bought 2 pounds of steak and found that 4 ounces were bone. What per cent was bone? What was the ratio of bone to meat?

5. The money taken in at a church social was $150. The expenses were $25. What per cent of the receipts were the expenses?

6. James paid a stage driver 75 cents for a ride of 5 miles. His rich uncle hired an automobile to take him over the same trip a few days later and paid $2.25 for it. What per cent of his uncle's payment was the amount the stage driver charged James?

7. An errand boy received 15 cents for delivering a package worth $1.80. What per cent of the value was the expense of delivery?

8. What per cent of a yard is a foot? 9 inches?

9. A baseball team won 125 games out of 200 games played. What per cent did it win?

10. A gas bill of $3.70 was reduced $.37 because paid within 5 days. What was the per cent of reduction?

More Algebraic Assistance — Rate Per Cent by a Formula. — When Helen solved the problem that her father gave her, she divided 23 (the percentage or part) by 25 (the base or whole) and thus obtained .92 or 92% (the rate or fractional part). She was really using this rule:

To find the rate when the percentage and the base are given, divide the percentage by the base.

That is, $\textit{rate} = \textit{percentage} \div \textit{base}$

Or $r = \frac{p}{b}$

Note that we express division by the fraction $\frac{p}{b}$ as well as $p \div b$. Thus $\frac{30}{100} = .3 = 30\%$, or $\frac{20}{5} = 4 = 400\%$.

Testing the Formula Orally

1. $\frac{25}{125}$ = what per cent?

2. $\frac{8}{24}$ is equivalent to what per cent?

3. $\frac{5}{6}$ of 84 is what per cent of 84?

4. What per cent of 36 is 6? of 56 is 7? of 48 is 6? of 72 is 12? of 96 is 32?

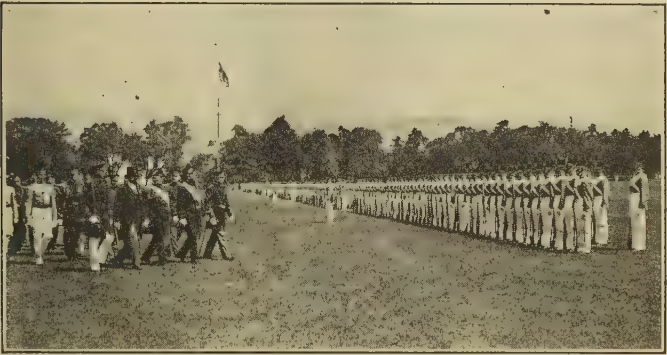
Making Algebra Coöperate with Arithmetic to Do Your Work

Using the above formula, solve each of the following:

1. $24 is what per cent of $120?

2. 27 days is what per cent of a 30-day month?

3. A company of 200 men is what per cent of a regiment of 1200 men?



West Point Cadets Being Reviewed by the President

4. In a school of 150 girls, 12 wore their hair long. What per cent of the girls had bobbed hair?

5. What per cent of a year is 3 months?

6. What per cent of 72 is 108?

Solution: $\frac{108}{72} = 108 \div 72 = 1.5$, which equals 150%. Note that the percentage here is the larger number.

7. What per cent of 40 is 50? of 50 is 125? of 48 is 60? of 32 is 96? of 56 is 98?

8. What per cent of 48 is 12? of 12 is 48? of 64 is 32? of 32 is 64?

9. What per cent of 18 is 6? 18 is what per cent of 6?

10. Of my class of 42, 7 pupils are girls. What per cent are girls? What per cent are boys?

11. 4 is what per cent of 400? 40 is what per cent of 400? 400 is what per cent of 400? 800 is what per cent of 400?

12. A boy having $32 spent $4. What per cent of his money did he spend? What per cent had he left?

13. A girl paid $2 of the total expenses, $16, of a camping party. What per cent of the total did she pay?

14. The expenses of a firm one year were $1200. Their sales were $6000. What per cent of their sales were their expenses?

15. Some freight was sent over five railroads, a total distance of 800 miles. The first road carried it 40 miles. What per cent of the total freight charge should it receive?

Finding the Base or Whole. — One day Allan heard his father say that 5% of the men employed in his factory were ill. Later he heard him tell a neighbor that 8 men were ill. Allan thought from this that he could find out how many were employed in the factory. So he said "$5\% = \frac{1}{20}$."

If $\frac{1}{20}$ of the number of men is equal to 8, then there must be 20 × 8 or 160 men in the factory. Was he right?

Another method, known as the x method or algebraic method, is shown you in Example 2 on the next page.

Illustrative Example 1. 18 is 9% of what number?

Solution: $9\% = \frac{9}{100}$ (or 9 hundredths)
$\frac{9}{100}$ (or 9 hundredths) of some number is 18.
$\frac{1}{100}$ (or 1 hundredth) of that number is 2.
$\therefore$ That number is $100 \times 2 = 200$.

Illustrative Example 2. 20% of a number is 16. Find the number.

Solution:

$$\begin{aligned} \text{Let } x &= \text{the number} \\ \text{Then } \tfrac{1}{5}x &= 16 \qquad (20\% = \tfrac{1}{5}) \\ x &= 16 \div \tfrac{1}{5} = 16 \times 5 = 80 \end{aligned}$$

This solution is by algebra. See how much easier it is. The New Mathematics aims to simplify all our work.

Oral Practice in Finding the Base or Whole

1. If $\frac{3}{4}$ of a number is 12, what is the number? *Hint:* Let $x =$ the number. Then $\frac{3}{4}x = 12$, etc.

2. If $33\frac{1}{3}\%$ of a number is 15, what is the number?

3. If 80% of a number is 20, what is the number?

4. 60% of a number is 24. What is the number?

5. $16\frac{2}{3}\%$ of John's earnings one week was $2. How much did he earn in the week?

Written Practice in Finding the Base or Whole

1. George made $2.50 one week selling papers. If this was $12\frac{1}{2}\%$ of the cost, what was the cost?

2. A real estate agent received 5% of all of the collections that he made for rent. He received $12 for collecting the rent on one house for one year. How much rent did he collect from that family?

3. A book agent sold books and received 20% of the selling price of the books as his pay. If he earned $150 one summer, how many dollars worth of books did he sell? How much did the publishers receive for the books after paying the agent?

4. At a certain entertainment the expenses were $45.50. If this was $16\frac{2}{3}\%$ of the amount taken in, how much money was taken in at the entertainment? How much money did they make?

5. Mr. Rogers is a gardener and raises tomatoes. After he had gathered them, 8 baskets, which was $2\frac{1}{2}\%$ of his crop, spoiled. How many baskets had he at first?

6. A tailor sold a suit for $16\frac{2}{3}\%$ less than it cost him, owing to the fact that a man for whom he made the suit moved away and did not take it. The tailor lost $6.50 on the suit. What did it cost him?

7. In the Roosevelt Junior High School 40% of the pupils were in the first year. There were 300 pupils in the second and third years. How many were there in the school?

The Review System

1. State the quotient mentally in each of the following:

$3.6 \div 1.2$
$.45\overline{)90}$
$.45\overline{)9.0}$

$.8\overline{)2.4}$
$.24 \div .8$
$.15\overline{)75}$

$.15\overline{)7.5}$
$.62\overline{)12.4}$
$.62\overline{)1.24}$

2. The *reciprocal* of a number is 1 divided by that number. What is the reciprocal of each of the following:

2, 4, 6, 8, 10, 15, 100, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{3}{4}$?

3. Change each of the following to kilometers:

25 miles. 30 miles. 35 miles. 40 miles. 50 miles.

4. $\frac{3}{4}\%$ ($\frac{3}{4}$ of 1%) of $1600 = ?

5. If you knew the total cost of some boxes of apples and the cost of each box, how could you find the number of boxes?

Finding the Base by Formula. — If we know that $8 \times 12 = 96$, do we know that $\frac{96}{8} = 12$ or $\frac{96}{12} = 8$?

The product of 6 and some number is 72. What is that number? How did you obtain it?

In a similar manner 8% of 600 or $.08 \times 600 = 48$. Therefore $\dfrac{48}{.08} = 600$. That is, since we multiply the base by the rate to get the percentage we must

Divide the percentage by the rate to obtain the base.

That is, $$base = \frac{percentage}{rate} \quad \text{or } b = \frac{p}{r}$$

Illustrative Example 1. 20 is 16% of what number?

Solution: 20 is the **percentage** or ***p***. .16 is the **rate** or ***r***.

$$\text{Then } b = \frac{20}{.16} = 125. \qquad \begin{array}{r} 125.\\ {}_{\times}16\overline{)20_{\times}00.}\\ \underline{16}\\ 40\\ \underline{32}\\ 80\\ \underline{80} \end{array}$$

Note that we cannot divide $16\%\overline{)20}$. Whenever we are using ***r*** (the rate), we must change it to a decimal or common fraction before it is possible to divide or multiply by it.

Illustrative Example 2. 75% of a number is 15. Find the number.

Solution: 75% is the *rate* or ***r***. 15 is the *percentage* or ***p***.

$$75\% = \tfrac{3}{4} \qquad\qquad \text{Or} \quad .75\,b = 15$$

$$15 \div \tfrac{3}{4} = 15 \times \tfrac{4}{3} = 20. \qquad\qquad b = 15 \div \tfrac{3}{4} = 20.$$

Testing the Formula Orally

1. 25% of a number is 24. What is the number?

2. 60% of a number is 45. Find the number.

3. 66⅔% of a number is 64. What is the number?

4. 80% of James's earnings in a week were $12. How much did he earn in the week?

5. .08 of a number is 16. What is the number?

Testing the Formula in Everyday Problems

Using the above formula, solve each of the following:

1. 12 is 8% of what number?

2. 24 is 6% of what number?

3. If 15 is 5% of a number, what is it?

4. What number is that of which 48 is 12%?

5. George spent 9 cents one week. This was 30% of his weekly allowance money. How much was his weekly allowance?

6. If Henry spent $6 for books, pencils, pads, notebooks, etc., when he entered school and this was 66⅔% of the money that his father gave him for these things, how much did his father give him? How much had he left?

7. At a special dress sale a certain dress was sold for $24. If this was 75% of the former price, what was the former price?

8. If a man pays $600 a year for board and room and this is 60% of all his expenses, what are all of his expenses? How much are his other expenses?

9. Merton missed 3 words or 15% of all the words in a spelling test. How many words were there in the test? How many did he spell correctly?

10. If a man receives $600 for the use of some money which he had loaned out to another man for one year and this was 6% of the amount loaned, how much money was loaned?

I. R. B. RECORD OF THE ALGEBRAIC FORMULAS OF CHAPTER VIII

Turn back through the pages of this chapter to find the formulas which you have been using. Then copy them into the records of your I. R. B. Test yourself to see whether you have committed them to memory.

"C. C. AND C. C."

If your teacher agrees and you wish, you may do the following exercise by the Coaching Committee plan.

Miscellaneous Review

1. Write the following per cents as hundredths and reduce them to simplest form:
6%, 50%, 75%, 15%, 80%, 25%, 2%, 20%, 8%, 4%, 10%, 40%.

2. Write each of the above in decimal form.

3. $12\frac{1}{2}\%$ may be written .125. This equals $\frac{125}{1000} = \frac{1}{8}$. Write as decimals: $4\frac{1}{2}\%$, $7\frac{1}{2}\%$, $8\frac{1}{4}\%$, $6\frac{1}{4}\%$, $24\frac{1}{2}\%$, $35\frac{1}{5}\%$.

4. Change each of the above decimals to common fractions in their lowest terms.

5. Find 8% of \$150; $3\frac{1}{2}\%$ of \$120; $4\frac{1}{2}\%$ of \$850; 7% of \$840.20.

6. 60 is 10% of what number? 12 bu. is 6% of what? \$3 is 5% of how many dollars? 5 inches is 5% of what?

7. On a test in mathematics Mary solved correctly 9 problems out of 12. What per cent should her paper be marked?

8. Fred wanted a football that was marked \$5. One day later it was marked down 20%, so he bought it. What did he pay for it?

9. In 1925 Harry and John each weighed 96 pounds. One year later they were weighed again. It was found that Harry had gained $16\frac{2}{3}\%$ and John had increased his weight by $\frac{1}{4}$. What did each weigh then? Draw a graph to show the increase in weight.

10. Change each of the following per cents to common fractions in their lowest terms:

$12\frac{1}{2}\%$, $8\frac{1}{3}\%$, $16\frac{2}{3}\%$, $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, $37\frac{1}{2}\%$, $62\frac{1}{2}\%$, $87\frac{1}{2}\%$, $6\frac{1}{4}\%$, $83\frac{1}{3}\%$.

11. At the rate of six apples for a quarter, how much will one and one-half dozen cost?

12. If a two and one-half pound box of candy cost $1.50, how much is that per pound?

13. If you get a 20% reduction on a suit marked $30, how much will it cost you?

14. If the distance between two places is 100 miles and you were to represent this on squared paper by 1 inch in length, what per cent of the distance would one small square (10 to the inch) represent? 3 small squares? 8 small squares? $4\frac{1}{2}$ small squares? 6 small squares would mean how many miles? 7 large squares? $2\frac{1}{4}$ large squares?

Word Problems

1. If you knew the multiplicand and the product, how could you find the multiplier?

2. If you knew the price of one pound of sugar and the cost of a certain number of pounds of it, how could you find the number of pounds?

3. If you knew the percentage and the rate, how would you find the base?

4. If you knew the base and the rate, how would you find the percentage?

5. If you knew the percentage and the base, how would you find the rate? Give the algebraic formulas for examples 3, 4, and 5.

6. If you knew a man's yearly salary and the rate of increase that he received for the next year, how could you find his next year's salary?

7. How would you work example 6 if the word "decrease" replaced the word "increase"?

8. If you knew the scale of miles to the inch on a map, how would you proceed to find the distance between two places on that map?

9. If you knew the marks of every member of your class in a mathematics test, how could you find their average mark? Perhaps your teacher will give you these marks sometime to test you on this question.

10. If you knew the average sales of ten clerks in a store for one day, how could you find the total sales in that store on that day?

REVIEW TRIAL OF I. R. B. TESTS, CHAPTER VII

Take the third trial of the six I. R. B. tests of Chapter VII. Persist in your trials until you have improved the scores already made in the First and End-of-Chapter columns. Then enter your third scores in the Review Trial column.

Percentage Review

The following examples should be worked mentally when possible in order to save time and develop mental work. Any omissions are to be filled in by the pupil.

I. R. B. Tests [3, 4, 5, 6]. Use parts *a*, *j*, *k*, and *l* for I. R. B. Test records. Your Steering Committee should determine a standard time for each test. Enter your time and accuracy scores in the First Trial column.

[3]

(a)

3% of 600 =
8% of 350 bu. =
11% of 950¢ =
15% of $840 =
25% of 648 gal. =

(b)

$16\frac{2}{3}\%$ of 540¢ =
25% of 1648 in. =
75% of $1800 =
30% of 6000 yd. =
$87\frac{1}{2}\%$ of 1640 =

(c)

$\frac{1}{4}\%$ of $60,000 =
$\frac{1}{5}\%$ of 80,000 =
$\frac{1}{10}\%$ of 505,000 =
$\frac{3}{4}\%$ of 16,000 ft. =
$1\frac{1}{2}\%$ of $60,000 =

(d)

150% of 20,000 = 1.5 times 20,000 =
160% of 56,000 =
200% of 2 =
250% of 16 in. =
300% of 50 lb. =

(e)

10 feet = 50% of —— feet.
50 inches = $33\frac{1}{3}\%$ of —— inches.
48 cents = $66\frac{2}{3}\%$ of —— cents.
63 pounds = $37\frac{1}{2}\%$ of —— pounds.
300 dimes = 60% of —— dimes.
880 yards = 50% of —— yards.

(f)

6 feet = what % of 12 feet?
1 dime = what % of 1 half-dollar?
2 ounces = what % of 1 pound (avoir.)?
8 games = what % of 12 games?
50 cents = what % of 10 dollars?
18 inches = what % of 3 feet?

(g)

.5% ($\frac{5}{10}$ of 1%) of 6000 =
.005% of 6,000,000 =
.6% of $800,000 =
.12% of 2,000,000 =
.375% of 12,000,000 =

(h)

.54 times 462 =
4.06 times 8.63 =
.12 times 42.1 =
12.12 times 8 =
.1212 times 8 =

(i)

.01)40 .001)400 .006)1212 .9)549 .012)243648

[4] (*j*)

Base = \$400. Rate = 5%. Percentage = ?
Base = 1200. Rate = 8%. Percentage = ?
Percentage = 25. Base = 125. Rate = ?
Percentage = 12. Rate = 25%. Base = ?
Percentage = 24. Rate = 75%. Base = ?

[5] (*k*)

\$36 increased by 25% = ?
\$20 increased by 40% = ?
56 mi. increased by 87½% = ?
80 increased by 80% = ?
\$500 increased by 20% = ?

[6] (*l*)

\$15 decreased by 33⅓% = ?
\$48 decreased by 16⅔% = ?
80 decreased by 60% = ?
72¢ decreased by 37½% = ?
1000 decreased by ½% = ?

(*m*)

Team A won 8 games out of 12 played. What per cent did it win?

Mary solved 10 examples correctly out of 12. What per cent is that?

James lost 3 sets out of 5 played. What per cent did he win?

Harry lost 6 cents of the 48 cents which he had. What per cent of his money did he lose?

(*n*)

75 is what per cent of 225? of 450? of 75? of 15?
What per cent of 13 is 6½? 300 is what per cent of 3?
.5 is what per cent of 1.5? of 4? of 2.5?

(*o*)

Complete the following:

BASE GAMES PLAYED	PERCENTAGE GAMES WON	RATE PER CENT OF GAMES WON
8	5	
24	18	
20	14	
154	98	
154	104	

Review of Per Cents

I. R. B. Test [7]. Write or state as many correct answers of the following as you can in 8 minutes. Enter your scores of time and accuracy in the First Trial column.

1. $12\frac{1}{2}\%$ of \$3.20.
2. 5% of \$400.
3. $\frac{1}{2}\%$ of \$500.
4. 150% of 300.
5. $\frac{1}{4}$ of a no. = ? % of the no.
6. $\frac{3}{4}$ of a no. = ? % of the no.
7. 16 = ? % of 32?
8. 8 = ? % of 4?
9. 15 is $33\frac{1}{3}\%$ of ?
10. 1 is $\frac{1}{2}$ % of ?
11. \$36 + 25% of \$16 = ?
12. \$30 − 50% of \$10 = ?
13. \$5 − $\frac{1}{4}\%$ of \$400 = ?
14. $16\frac{2}{3}\%$ of 24¢ from 25¢ = ?
15. $\frac{1}{2}\%$ of \$100 from \$1 = ?
16. 500% of \$3 plus \$20 = ?
17. 1000% of \$1 − \$5 = ?
18. $5\frac{3}{4}\%$ of \$2000 = ?
19. \$24 is $37\frac{1}{2}\%$ of ?
20. 24¢ = $\frac{3}{4}\%$ of ?

Retrospect

1. A real estate agent received 3% of the selling price for selling some property. If he received \$9000 as his pay, what was the selling price of the property?

2. What were the sales of a traveling salesman if he received as his pay \$846.50, which was $12\frac{1}{2}\%$ of his total sales?

3. A real estate agent bought some land for \$9650. If it increased in value 35% at the end of a year when it was sold, how much did he make on it? (Interest is not to be considered.) Had it decreased 25%, for what would he have had to sell it?

4. Another real estate speculator bought some property for \$12,000 and found that it was decreasing in value due to manufacturing interests moving away from that city, so he had to sell it at the end of a year at a decrease of 14%. How much did he lose? What was the selling price?

5. What per cent would Frank receive if out of 50 examples he did 40 right? 30? 45? 42?

6. A real estate agent received $120 for collecting rents. If he received 5% of all of his collections as his pay, how much rent did he collect?

7. Another real estate agent collected $1675 in rents. If he received 4% of his collections as his pay, how much did he earn?

8. A tailor made 80 yards of cloth into suits. If he used, on the average, $6\frac{2}{3}$ yards for each suit, how many suits did he make?

9. What per cent of a rod is 3 feet 8 inches? *Hint:* First change each to feet.

10. One mile of railroad track was to be laid with rails which weighed 80 pounds to the yard. If 1000 pounds of such rails cost $25.75, how much would it cost to lay the tracks?

11. A railroad train is running at the rate of 60 miles an hour. How many feet per second is it traveling?

12. Which is greater and how much greater, 5.8% of $1500 or 4.6% of $2000? What is the ratio of the smaller to larger?

13. A man invested $\frac{2}{3}$ of his money in real estate, $\frac{1}{3}$ of the remainder in stocks, and still had $6000 in the bank. How much property had he in all?

14. A merchant had a suit marked $32.40 before a sale. During the sale he sold it for $24.30. How much did he reduce it? What per cent did he reduce it on the marked price?

15. Mr. Scott was earning $2600 a year. His employer increased his salary to $2860 the following year. What per cent was he raised? How much per week was he raised?

16. My automobile tire is 106 inches in circumference. If the wheels have turned 10,000 times, how many miles have I traveled?

17. 22.8 pounds of butter fat were obtained from 480 pounds of milk produced by a very good Jersey cow. Find the per cent of butter fat.

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER VIII

In this chapter you have begun to study some of the *mathematics of business.* You have discovered that percentage is a very common tool in business. You have practiced the interchanging of equivalents in fractions, decimals, and per cents. You have developed some skill in selecting the equivalent which is the easiest tool to use. You know how business men find per cent of increase or decrease. You have learned how to use rate per cent, base, and percentage and how to find one when the other two are known. You have used formulas which are the beginnings of Algebra. Thus you have learned how the several parts of your New Mathematics coöperate to solve your own and others' everyday problems of business.

Now take once more the seven I. R. B. tests of this chapter. Try to improve your First Trial record. Work independently out-of-school-hours. Record your best time and accuracy scores in the column headed End-of-Chapter Trial. Remember the greatest test of all is the trial of your trustworthiness. If you pass this test with a 100% record, you need not be disturbed overmuch by lower scores in your mathematics skill. They will improve with practice.

CHAPTER IX

FIRST STEPS IN EVERYDAY BUSINESS. MARGIN

The Language of Business. — We go into business to make money. To tell how much we make we must know not only costs and selling prices, but also what business men call *overhead*, or operating expense. Overhead includes rent, salaries, light, heat, telephone, stationery, insurance, and all expenses connected with a business.

For instance, in a shoe business, if you buy a pair of shoes for $3.00 and sell them for $5.00, you do not make a profit of $2.00. That $2.00 is your *margin*, the difference between buying and selling price. It is sometimes called *gross profit*. Your *profit* is what is left of the margin after a certain amount is deducted for overhead. If your overhead averages $1.20 for each pair of $5.00 shoes sold, your profit is $.80. This is sometimes called *net profit*.

Per cent of margin is usually figured on the selling price. When the per cent is figured on the cost, it is called *mark-up*.

Discount. — One of the commonest practices in business is called *discount*. This is a percentage of the regular or *list price* that is taken off because the customer pays cash, or buys a large quantity, or for some other reason. *Trade discount* is the allowance to the wholesaler or retailer to pay him for handling and selling the goods. Discount is computed just as we computed percentage on page 189 of the preceding chapter.

In business, discount is often expressed simply by "off." When you say: "How much off for cash?" you mean: "How much discount for cash?" "Anything off if I buy a dozen?" means: "Is there any discount if I buy a dozen?"

Illustrative Example 1. A merchant had a certain suit marked at $40. On account of poor sales he decided to reduce all of his goods 20%. What was the new price of that suit?

Solution: 20% or ⅕ of $40 = $8.
$40 − $8 = $32.

Or
5)$40 List price
8 Discount
$32 New price.

Therefore the new price was $32.

Illustrative Example 2. A dealer in straw hats had two kinds, one marked $4 apiece and the other $3.50 apiece, August 1, when the season for straw hats was nearly over. he offered these hats at a discount of 40%. What was his new price for each kind of hat?

Solution:
40% or ⅖ of $4.00 = $1.60, discount
$4.00 − $1.60 = $2.40, selling price.

Or
.80
5)$4.00
2 × .80 = 1.60
$2.40

40% or ⅖ of $3.50 = $1.40, discount
$3.50 − $1.40 = $2.10, selling price.

Or
.70
5)$3.50
2 × .70 = 1.40
$2.10

Hence his new prices were $2.40 and $2.10.

I. R. B. Test [1]. Your Steering Committee and Teacher (ex officio) should determine a standard time for this test and for the other I. R. B. tests of this chapter when you take the First Trials. Prepare chart for Chapter IX (4 tests). Enter your First Trial scores. Complete this test by filling in the blanks below.

GOODS MARKED	WERE SOLD AT A DISCOUNT OF	DISCOUNT	SELLING PRICE
$150	20%		
$180	$33\frac{1}{3}$%		
$2	10%		
75¢	40%		
16¢	$12\frac{1}{2}$%		
24¢	$16\frac{2}{3}$%		
$3	5%		
$6	8%		
$16	6%		
$40	$4\frac{1}{2}$%		

DRAMATIZING DEALS IN DISCOUNTS

Ten pupils appointed by Steering Committee and Teacher are to act as salesmen; and one of the following ten problems is to be assigned to each salesman. The salesmen choose one buyer each. Then each salesman in order is to give his problem to his buyer. The buyer works on blackboard the problem assigned — one problem on the blackboard at a time. When each buyer has completed the blackboard work, his salesman explains the solution to the class.

Class members not participating check work of buyers and explanations of salesmen. The Steering Committee assists only when buyer, salesman, and class fail to get the right solution.

Discount Drill

1. George's father wanted to get George a bicycle which he saw in a store window. It was marked $60 and he felt he could not afford it. Some weeks later the merchant had to close out his goods at a reduction, so he offered all of them at 30% discount. Then George's father bought the bicycle. How much did he pay for it?

George and His Bicycle

2. Mrs. Seeley wanted to buy a set of dining room furniture which was sold on the instalment plan for $325. She preferred to pay cash for it and so received a discount of 10%. How much did she save by paying cash? How much did the set cost her?

3. A shoe merchant bought some goods, which were billed to him at $95.64. If he received 2% discount for cash, what would be the net amount in case he paid cash?

4. A book dealer bought books listed at $240 and received

25% discount on them. How much was his discount? How much was the net price to him?

5. A certain gas company allowed 10% discount on all bills paid before the 5th of the month. How much would Mrs. James save on her bill of $3.80 if she paid within 5 days? How much would she have to pay?

6. Complete the following:

Goods Marked	Were Sold at a Discount of	Discount in Money	Selling Price
$250	30%		
$75	25%		
80¢	15%		
$1.50	10%		
50¢	28%		

7. An article was listed at $28 but is finally sold at a discount of 12½% from the list price. If the buyer, who is a retailer, sells it for $29.40, what per cent does he make on the cost to him? What per cent does he make on his selling price?

8. A retailer bought an article at a discount of 16⅔% from the list price of $24. How much did it cost him? If he sells it for 33⅓% gain on his cost, what is the retailer's selling price?

9. A certain kind of men's shirts selling regularly for $2.40 were sold at a discount of 12½% during a sale. How much would the shirt cost during the sale?

10. A certain radio set was sold for $150 on a partial payment plan but a discount of 10% was offered for cash. What would be the cash saving? the cash selling price?

Finding the Rate of Discount. — Sometimes a business man knows the amount of the discount, but wants to find the rate. This is shown by the following

Illustrative Example. Henry Welsh bought for $25 a suit that had been marked $30. Find the rate of discount.

Solution: Henry received a discount of $5 on the former marked price.

$$\frac{5}{30} = \frac{1}{6} = 16\tfrac{2}{3}\%, \text{ rate of discount.}$$

Hence to find the rate of discount we have the following rule:

1. *Subtract the selling price from the marked price to obtain the discount.*
2. *Divide the discount by the marked price to obtain the rate of discount.*

Completion Test

I. R. B. Test [2]. Enter First Trial scores. Complete the following:

MARKED PRICE	SELLING PRICE	CASH DISCOUNT	RATE OF DISCOUNT
$120.	$100.		
60.	55.		
3.	2.75		
6.	4.		
12.50	11.		
10.50	8.		
21.20	18.55		
450.	405.		
72.60	60.50		
8¢	6¢		

Pencil-Paper Practice

1. A suit of clothes had been marked $35 but on account of poor sales it was sold for $25. What was the per cent of discount?

2. A piano was marked at $450 but was sold for $300. Find the per cent of discount.

3. A lady bought for $5 a pair of shoes which had been marked $7.50. What per cent discount did she receive?

4. Mr. Slocum bought a lot for $1000 and built on it a house costing $9000. He then sold the property for $9500. What was the rate of discount on his total cost?

Mr. Slocum's House

5. A second-hand automobile had been marked $850 but was bought for $695 cash. What per cent of discount did the cash buyer receive?

6. At a cash sale a lady's suit was sold for $28.80 which had been marked at $36. Find the rate of discount.

7. What rate of discount is received if I buy an article for $11 which was marked $16?

8. Which of the following offers the best discount and by what per cent?

Marked price $50.	Selling price $42.
Marked price $67.50.	Selling price $54.
Marked price $24.	Selling price $20.

9. See if you can find from store windows or newspaper advertisements examples of the type of example 8 and bring them to class with you.

10. Give examples of goods which are likely to be sold at a discount at certain times of the year and give your reasons for this.

Finding the Margin of Loss or Decrease. — Discount is a decrease in the selling price. On page 196 we figured the per cent of decrease. In business if the selling price is less than the buying price, there is a margin of loss or decrease. This is figured by decimals just like discount.

Illustrative Example. George bought a sled for $2 which he afterwards sold at a loss of 34%. How much did he lose and for what did he sell it?

Solution: $.34 \times \$2 = \$.68$, his loss.
$\$2.00 - \$.68 = \$1.32$, his selling price.

Because this is like discount we shall need very little extra practice in it.

Figuring Losses

1. $37\frac{1}{2}$% loss on a cost of 16 cents is how much?

2. If the cost is $8.00 and the loss is 8% of the cost, what is the loss?

3. A merchant had $15,000 worth of goods in his store. A fire caused a damage of 25%. What was the loss if he had no fire insurance?

4. How much will you lose if you sell a piano, that cost you $350, for 14% less than it cost you?

5. A leather merchant bought $20,000 worth of goods in war time. Later he had to sell them at a loss of 15%. How much did he lose?

Finding the Margin of Increase or Advance. — On page 194 we studied the per cent of increase. In business when we know the cost and the per cent of increase, it is easy to find the amount of the margin of advance or increase.

Illustrative Example. Harry bought a jackknife for 60 cents and sold it at an advance of 15% on the cost. How much did Harry receive for the knife?

Solution: .15 × \$.60 = \$.09, advance in money.
\$.60 + \$.09 = \$.69, his selling price.

Finding Increases Mentally

1. $12\frac{1}{2}\%$ advance on a cost of \$24 is how much?

2. $33\frac{1}{3}\%$ advance on a cost of 36 cents is how much?

3. If you buy an article for 3 cents and sell it at an advance of $66\frac{2}{3}\%$ of the cost, what is your advance in money? For what do you sell it?

4. If a young man bought an old automobile for \$125 and sold it at an advance of 10%, what was the amount of increase?

5. A man bought a house one day for \$8000 and sold it two weeks later at an advance of 25%. What did he make?

Oral Estimates

Check your estimates on paper.

1. What margin of increase did Martin receive if he bought a box of candy for \$1.25 and sold it for \$1.50?

2. A shoe merchant bought some shoes for \$4.00 a pair. For what must he sell them to make a margin of 25% on the cost? of 35%? of 40%?

3. Mr. Murray sold at an increase of 15% a horse that cost $200. How much money was gained? What was the selling price?

MR. MURRAY'S HORSE

4. Mr. Emory bought 1000 yards of dress goods for 62½ cents a yard and sold them at an advance of 30% on the cost. What did he receive for the goods?

5. A man bought some fat cattle for $520 and sold them for beef at an advance of 25% on the cost to him. How much did he receive for them?

6. A dealer, who has $100 to invest, finds that he can make 28% if he invests it in turkeys, or 32% if he invests it in chickens. How much more will he make if he buys chickens than turkeys?

Finding the Per Cent of Mark-up or Per Cent of Loss on the Cost. — If Harry Smith should buy a kite for 80 cents and sell it for $1.00, he would make 20 cents on the deal. This is his margin. If Harry had no "overhead" expenses, this would also be his profit. This difference, if reckoned as a per cent of the cost, is called *mark-up*. In the case of Harry's kite he made 20 cents on an investment of 80 cents. He, therefore, made a mark-up of $\frac{20}{80}$ or $\frac{1}{4}$ or 25% on his investment.

Sometimes dealers are so unfortunate that they have to sell goods for less than cost. In this case there is a *loss* which is also usually computed as a per cent of the cost.

Illustrative Example. During the War a grocer bought sugar at 12 cents a pound. Soon afterwards the War ended, prices dropped, and he was obliged to sell it at 10 cents a pound. What per cent did he lose on his first cost of the sugar?

Solution: He lost 2 cents on each pound which cost him 12 cents. Hence he lost $\frac{2}{12}$ or $\frac{1}{6}$ or $16\frac{2}{3}\%$ on the cost.

COACH AND PLAYER PRACTICE IN ESTIMATES

The Steering Committee should divide the class into coaches and players. The committee with the teacher's assistance should decide which of the 15 examples beginning on page 226 are adapted to estimates; examples omitted will be used for another purpose.

Each coach and player should write an estimate for each example selected for estimating. Then the player should work the example on paper and the coach should check the answer; in case of doubt call on the Steering Committee. Score one for the estimate of player or coach of each couple which is nearer to the correct answer. In the same manner the rest of the estimates should be written, worked, checked, and scored.

At the end of the estimate work the Steering Committee should total the scores made by all coaches and the scores made by all players. Before deciding whether coaches or players win the game the Steering Committee may give the players a handicap.

After the contest the Steering Committee should work on the blackboard the problems not selected for estimating.

Oral Work Saves Business Time

1. $3 is what per cent mark-up on a cost of $24?

2. If the cost is 15 cents and the increase is 3 cents, what per cent mark-up is this on the cost?

3. A loss of 3 cents on a cost of 18 cents is —— per cent loss.

4. The cost is 25 cents. The loss is 10 cents. Find the loss per cent on the cost.

5. A man buys a cow for $80 and sells her at an increase of $10. What is his per cent of mark-up?

Exercise

1. A man bought a house for $10,000 and sold it at an advance of $1000. What was his per cent of mark-up ?

2. Mr. Berry sold a mill, which cost him $25,000, for $5000 less than he paid for it. What per cent did he lose?

A Group of Mills

3. Copy the following form and complete it by filling in all missing spaces:

Cost	Selling Price	Mark-up	Per Cent Mark-up	Loss	Per Cent Loss
$100	$120	$20			
80	60				
60	70				
120	140				
300	250				
180	200				
50¢	65¢				
75¢	50¢				

4. Alice Smith's father owns a grocery store. Last month he bought 60 dozen eggs at 40 cents a dozen, and sold them at 48 cents a dozen. What was his margin in dollars?

5. A fruit dealer bought oranges at 48 cents a dozen and sold them at 60 cents a dozen. What per cent margin did he make?

6. A huckster bought 100 bushels of apples from a farmer for 50 cents a bushel, and sold them in the market at 65 cents a bushel. What per cent of increase did he make on the cost of all? on the cost of one bushel?

7. A merchant bought two dresses, one for $24 and the other for $28.50, and sold them for $32 and $40 respectively. What per cent was his margin on the cost of each dress?

8. If a newsboy buys his papers at 1½ cents apiece and sells them at 2 cents apiece, what is his per cent of mark-up?

9. What is the per cent of loss if an article, which cost $8, is sold for $6?

10. A merchant bought 370 yards of cotton cloth at 8 cents a yard and sold it all for $37.00. What was his per cent of margin?

11. What is the loss per cent on goods bought for $7650 and sold for $6575?

12. A boy had 42 cents in the morning but lost 7 cents on his way home from school. What per cent of his money did he lose?

13. An automobile cost $1500 but on account of a death in the family was sold for $1150 before it was used. What was the per cent of loss?

14. A suit of clothes cost the merchant $24.00. He was obliged to sell it for $20. Find the per cent of his loss.

15. James bought a new motorboat for $350 but on account of having to move a great distance away he sold it for $200. What per cent did he lose on the cost?

Computing Per Cent of Margin on the Selling Price. — Most business firms today compute their per cent margin on the selling price. For example, an unfinished article costs 20 cents. Work to put it into finished condition for sale and overhead expenses of selling bring the total cost to 90 cents. The Selling Price is $1.20. The margin between the first cost of the unfinished article and the selling price of the finished article is $1.20 − .20 = $1.00. But the net profit is $1.20 − .90 = .30. It is customary to compute the gain per cent on the *selling price*, or in this case $r = \frac{p}{b}$, $r = \frac{.30}{1.20} = \frac{1}{4} = 25\%$; therefore, the per cent of gain on the selling price is 25%.

Practically all large business houses now compute their per cent of gain on the selling price as shown in the above illustration. Large businesses like chain stores and department stores are run on a small net profit per article or on a small per cent gain on the selling price. Overhead expenses are figured on the selling price. Even calculating machines are adapted to these methods of modern business. Thus the practice grows to use the selling price as the basis for computing per cent gain, profits, commissions, overhead expenses, etc. A former practice of using the cost as the basis is being steadily abandoned.

So to find the margin per cent on the selling price we have the following rule:

Divide the difference between selling price and cost price by the selling price to obtain the per cent of margin.

Oral Work Saves School Time

1. Complete the following:

SELLING PRICE	MARGIN PER CENT ON SELLING PRICE	GAIN IN MONEY
$16	25%	
$20	20%	
$1	15%	
18¢	$16\frac{2}{3}$%	

2. Find the margin per cent on the selling price in the following:

COST	SELLING PRICE	MARGIN IN MONEY	MARGIN PER CENT ON SELLING PRICE
$10	$20		
$15	$20		
$18	$24		
$4.50	$6		
36¢	48¢		

3. A certain store sold $500,000 worth of goods one year and made 10% on the selling price. What were the profits?

4. An agent sold $125 worth of books and received 40% of the selling price for his work. How much did he receive? How much did the publishers receive?

5. A certain chain of stores sold about $215,000,000 worth of goods in a year. If their profits were 10% of their sales, what were their profits?

6. A dealer had an overhead expense amounting to 25% of the selling price. He paid $12.00 for a dozen pocket knives and sold the dozen for $16.00, adding 33⅓% to his cost. How much was his profit?

Estimate and Pencil-Paper Review

1. Complete the following as shown in the first part of the example:

5.36% of $60,000 = .0536 × $60,000 = $3216.
8.4% of $2,000 =
1.35% of $80,000 =
12.2% of $4,000 =
25.64% of 56,758 =
36.75% of 86,745 =

2. The selling price is 48 cents and the mark-up is 12½% of the selling price. Find cost and increase in money.

3. John received $25 a week but on account of poor business his employer told him at the beginning of the year that he would have to reduce his wages 10%. What would be John's new wage per week?

4. Mary worked 40 examples correctly out of 50 possible examples given in tests during a given month. The next month she increased the number correct 12½% over the number correct the previous month. How many did she get correct that month? What per cent grade would she obtain out of 50 examples given in the last month?

5. 1 liter is equal to 1 cubic decimeter and is about equal to 1 quart in American measure. How many liters are there (approximately) in 8 gallons 3 quarts? How many cubic decimeters?

Bargains. How Business Men Sell Goods on Two or More Discounts from the Marked (List) Price. — Sometimes a man offers two or more discounts for different reasons. When business men do this, we can buy at bargain prices.

Illustrative Example 1. A merchant sold eggs at 40 cents a dozen. He offered a 10% discount from this price if the buyer bought a crate (30 doz.) and an additional discount of 5% for cash. The owner of a lunch room bought a crate for cash. How much did it cost him?

Solution:
List price = 40 cents.
10% of 40 cents = 4 cents, first discount.
$.40 – $.04 = $.36.
5% of 36 cents = $.018, discount for cash.
$.36 – $.018 = $.342, price paid per dozen.
30 × $.342 = $10.26, price per crate.

Or

$$\begin{array}{r} 10)\$.40 \\ \underline{.04} \\ 20)\ .36 \\ \underline{.018} \\ \$.342 \\ \underline{30} \\ \$10.26 \end{array}$$

Note: When there are two or more discounts to be taken off, we must first take off one discount and then a second discount from what remains, etc.

The discounts may be taken in any order that you may select, and obtain the same result. For example, take 20% and 10% from $100.

(*a*) $100 – 20% of $100 = $80. $80 – 10% of $80 = $72.
(*b*) $100 – 10% of $100 = $90. $90 – 20% of $90 = $72.

Do not add the discounts together and then take off their sum. If you do this in the preceding example you get

$100 – 30% of $100 = $70.

Illustrative Example 2. A wholesale merchant offered an article at $180 list with discounts of 20%, $16\frac{2}{3}\%$, and 10%. Find the net amount of the bill.

Solution:

6)$180 List price
30 First discount
5)$150
30 Second discount
10)$120
12 Third discount
$108 Net amount

Note here that if you use the $16\frac{2}{3}\%$ first rather than the 20% your work is easier. Therefore choose the order in which you think the discounts will work best before you start. Review Aliquot Parts, page 36.

I. R. B. Test [**3**]. Enter First Trial scores. Work the following ten examples in your head.

1. Find the net amount of a bill of $240 at list with discounts of $33\frac{1}{3}\%$ and 25%.

2. A bill of $20 with discounts of 25% and 20% equals what net price?

3. $48 less discounts of $16\frac{2}{3}\%$ and $12\frac{1}{2}\%$ equals what?

4. 20% and $16\frac{2}{3}\%$ discounts on a list of $72 equals what net price?

5. A merchant paid a bill of $50 list with discounts of 25%, 20%, and 2%. What was the net amount of this bill?

6. $60 less discounts of 20% and $16\frac{2}{3}\% = ?$

7. $150 minus discounts of 10% and $33\frac{1}{3}\% = ?$

8. $500 less discounts of 25%, 20%, and 2% = ?

9. What is the net amount of a bill of $360 less discounts of 25% and $16\frac{2}{3}\%$?

10. $250 less discounts of 20%, 25%, and 2% = ?

DRAMATIZING BARGAINS

The Steering Committee (teacher ex-officio) should appoint two, three, or more pupils as needed for the buyers and sellers of each of the following 13 problems. Follow the plan for Dramatizing Deals in Discounts on page 217 in regard to the blackboard work. The sellers to whom are assigned problems 8 to 13 should make actual business transactions of the list prices and discounts given.

Written Exercise

1. A furniture dealer offered a certain piece of furniture for $240 with a discount of 20% and a second discount of 10% for cash. What would this article cost me if I paid cash?

2. I bought 100 yards of woolen cloth listed at $1.25 a yard at discounts of 20% and 15%. What did I pay for it?

3. One wholesaler offered a piano dealer a certain piano at $480 with discounts of $33\frac{1}{3}$%, 25%, and 5% off for cash. Another offered him the same piano at $500 with discounts of 40%, 20%, and 10% off for cash. Which cash offer is the better for the retail dealer and by how much? Which cash offer is the better for the wholesaler?

4. One radio dealer offered me a certain radio at $120 list price with discounts of 10% and 5% for cash. Another offered me the same radio for $150 list price with discounts of 30% and 2% for cash. Which offer is the better for me (the buyer)? for the seller? How much better?

5. A merchant buys a bill of goods from a wholesaler. The list prices total $274.36. The wholesaler gives him 25% and 2% off. Find the net amount of the bill to the merchant. The merchant then sells the goods at an average mark-up of $12\frac{1}{2}$% on his cost. How much is the merchant's margin? For what does he sell the goods?

6. What will it cost me to buy 500 bales of hay (each bale, 200 pounds) listed at $28 a ton, if I can get discounts of 10% and 8%? (1 ton = how many pounds?)

Taking in the Hay before Baling It.

7. Prove that a bill of $150 with discounts of 16⅔% and 20% gives the same as a list price of $150 with discounts of 20% and 16⅔%.

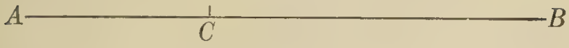
8. A dealer bought a large consignment of merchandise at a list price of $12,000. He received discounts of 20% and 10%. What is the difference between what he paid and what he would have paid if the discounts had been 25% and 5%?

Complete the following:

	List Price	Discounts	Net Price		List Price	Discounts	Net Price
9.	$60	25%, 20%		**12.**	$45	20%, 16⅔%	
10.	$90	33⅓%, 10%		**13.**	$.36	25%, 4%	
11.	$120	16⅔%, 5%		**14.**	$.42	25%, 16⅔%	

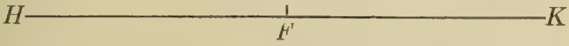
Oral and Written Review

1. Suppose line AB is 100 millimeters long.



Suppose AC is 35 millimeters long.
What per cent of the whole line is AC?
How long would a line be which is 56% of AB?

2. Suppose line HK is 100 millimeters long and HF is 50 millimeters long.



What per cent of HK is HF? of HF is HK?

3. Draw to a scale a line representing 100 miles. Now draw with the same scale a line representing 15 miles. Represent by a per cent, a common fraction, and a decimal the relation of the 15-mile line to the line of 100 miles.

4. A certain store marked each dollar's worth of goods down 25 cents. What per cent were they reduced?

5. State 1% of $800, $575, $490.35, $9.00, $18.80.

6. Give 4% of $1600, $2000, $525, $50.50, $235.

The Review Habit

1. In a certain factory 1.5% of the wages of each person was deducted for an insurance fund. If a boy received $20 a week salary, how much would be deducted from his salary in a year of 52 weeks?

2. A teacher had to contribute 4.6% of her annual salary toward a state teacher's pension fund. If her annual salary were $2200, how much would she have to contribute yearly?

3. How many times a number is 300% of it? 400% of it? 600% of it? 150% of it? 275% of it? 1000% of it?

WHOLESALERS' DISCOUNTS

When a retail dealer buys goods of a wholesaler, the latter frequently gives one or more discounts on the list prices of the goods and an additional discount for cash.

Have pupils volunteer to find actual problems of wholesalers' discounts. Pupils individually or in committees may secure these from actual cases. Ask parents, who are business men, or local merchants for actual cases of wholesalers' discounts. Get all the facts — what the list price was, what quantity was purchased, whether cash was paid, what discounts were given, and what the actual cost was. Make a problem of the facts of each case and bring the problem to your next class period.

When the class completes these original problems, have the Steering Committee appoint one pupil as a retail merchant and two pupils as traveling salesmen representing two wholesale houses. Let these three pupils conduct a buying and selling transaction of one of the problems selected by the Steering Committee for this purpose.

The merchant should give each salesman the bill of goods which he desires to purchase. The salesmen should offer their discounts, terms of payment, and describe quality, and manufacture of goods. The class should determine which of the different quotations is the lower.

The merchant should decide which goods to buy; he should base his decision upon discounts, terms or times of payment, and quality of goods. Repeat the dramatization for another original problem if the class wishes and time permits.

Finding the Marked (List) Price. — By paying cash for coal I received a discount of 10%, and so was able to buy coal for \$12.15 a ton. I wanted to know what the list price was so I calculated it as shown below.

Solution: First I let 100% represent the original or list price. 10% of the list price equals the discount. Hence 100% − 10% = 90% of the list price, and this is the selling price. Since \$12.15 represents 90% of the gross price we use the formula on page 205. $b = \frac{p}{r}$ and divide \$12.15 by .90.

```
           13.50
Thus .90)12.1500
          9 0
           315
           270
            450
            450
```

Or

$$\$12.15 \div \tfrac{9}{10} = 12.15 \times \tfrac{10}{9} = \$13.50.$$

Hence the list price was \$13.50 a ton.

We have the following rule:

To find the list or marked price divide the selling price by the result of subtracting the rate of discount from 100%.

I. R. B. Test [4]. Another completion test. Enter First Trial scores. Complete the following:

SELLING PRICE	RATE OF DISCOUNT	LIST PRICE
\$20	20%	
\$36	10%	
\$40	$16\frac{2}{3}$%	
\$130	15%	
\$225	8%	
\$75	$33\frac{1}{3}$%	
\$160	5%	
\$275	30%	
14¢	$12\frac{1}{2}$%	
25¢	$37\frac{1}{2}$%	

I. R. B. RECORD OF THE ALGEBRAIC FORMULAS OF CHAPTER IX

Copy from the textbook into your I. R. B. the formulas which you have used in this chapter. Change as many *rules* of the chapter into formulas as you can and copy these original formulas into your permanent record. Then test yourself to see whether you have committed the formulas and rules to memory.

Written Exercise

1. A Vermont farmer sold 100 gallons of maple syrup at a discount of 5% because the dealer bought a large quantity. The farmer received $190 for it. What was his list or marked price of all? of 1 gallon?

MAKING MAPLE SYRUP IN VERMONT

This picture shows President Coolidge's father carrying two sap buckets.

2. The selling price of a desk was $112. The discount was 30%. What was the marked price?

3. A man sold a house for \$8000. If he lost 20% on the cost, what did it cost him?

4. A boy spent 40% of his money and had \$120 left. How much had he at first?

5. The value of a building is \$3850. This is 23% less than the value of an adjoining building. What is the value of the adjoining building?

6. The expenses of a certain firm are 9% less than the receipts. If the expenses are \$7280, what are the receipts? What are the profits?

7. A man is now paying \$64 a month rent for a store. This is a saving of 20% on what he was previously paying. What was his old rent?

8. The selling price of some goods was \$9582.90. The discount offered on the marked price was 15%. What was the marked price?

9. The price of a suit is now \$36.80. This is 8% less than the price of the same suit last year. What was the price of the suit last year?

10. 27 pupils of a class were promoted at the end of the term. 10% of the class failed. How many were in the class? How many failed?

11. In a school of 1100 pupils exactly half were boys. Fifty of them tried for the football eleven. What per cent was this of the total enrollment?

REVIEW TRIAL OF I. R. B. TESTS, CHAPTER VIII

Try again the seven I. R. B. tests of Chapter VIII. Take tests independently. When you have improved the first two records, make the Review Trial entries in your I. R. B.

Selling on Commission. — Many people make their living by selling goods for some other person. Often they are paid a certain per cent of the selling price of the goods that they sell. The money which they receive for selling the goods is called *commission.*

Many people who sell butter, eggs, and produce of various kinds, do business on commission. So do traveling salesmen, such as book agents.

Some stores pay their sales people a certain wage and then an additional commission on their sales.

Illustrative Example: A real estate agent sold a house for $12,000. The agent received 5% commission for selling the house. How much commission did he receive?

Solution:

5% of $12,000 = .05 × $12,000 = $600, commission.

Note. The rate that such an agent would receive would vary in different cities and towns from 2% upwards.

An Oral Test

1. Find the money commission in each of the following:

SELLING PRICE	RATE OF COMMISSION	COMMISSION
$10,000	5%	
$15,000	3%	
$20,000	2%	
$720	12½%	

2. A boy took subscriptions for 140 magazines at $3 each. If he received 5% commission, how much did he earn?

3. An insurance agent received 20% commission on the premiums of the fire insurance which he collected. One month he collected premiums to the amount of $500. What was his commission?

4. A commission merchant sold apples to the amount of $656. If he received 10% commission, how much commission did he receive?

Making Money by Selling on Commission

1. A commission firm sold poultry for a man from the country to the amount of $256. If they received 12½% commission, how much was their commission for selling the poultry?

2. An automobile salesman received 15% commission for selling an automobile. How much commission would he receive on a car that he sold for $1500? for $1200? for $900?

3. A real estate agent received 3% commission for selling a house for $6000. How much commission would he receive? How much would the seller of the house receive?

4. A lawyer received 16⅔% commission for collecting a debt of $738. What was his commission?

5. A salesgirl received $10 a week and a commission of 1½% on her sales. How much did she earn in a week if she sold $256 worth of goods?

6. A traveling salesman received 12½% of all of the sales that he made. If he sold $3056.24 worth of goods in one month, how much did he earn? How much would his firm receive after paying him?

7. An auctioneer received 3% on all of the goods that were sold at a certain auction. If he sold goods to the amount of $1256.75, how much did he receive? How much did this net the seller?

8. If a radio agent received 20% of his sales, how much did he earn in a week preceding Christmas if his sales amounted to $1275?

9. A man received a position through an employment agency which charged a commission of 10% of his first month's salary and 5% of his salary for the next three months. If his position paid him $25 a week, how much commission would he have to pay the agency? How much would he have left the first month? the second month?

Estimate and Check Review

1. How much will it cost to insure a building for $8500 at 48¢ per $100?

2. A man sold a suit for $36. If he gained 20% on the cost, what did it cost him?

3. How much would you lose if you sold a pair of shoes, that cost you $4.50, for 15% less than they cost you?

4. If you sell a pair of shoes for $10.00 and your profit and expenses are $2.50, what per cent of the cost to you are the profit and the expenses?

5. Write the equivalent decimal for each of the following:

$1\frac{1}{2}\%$, 475%, $5\frac{1}{4}\%$, 125%, 55.6%, $6\frac{3}{4}\%$.

Buying on Commission. — Some people also buy goods for another person or firm on a commission basis. The method of calculating the amount of commission is the same as in the preceding section.

Making Money by Buying on a Commission

1. A real estate agent bought a house for me. He paid the seller of the house $9000. The agent charged me $2\frac{1}{2}\%$ for buying it. How much was his commission? How much did the house cost me after paying him his commission?

2. An agent bought for a merchant $6000 worth of cloth and charged 2% of the cost of the goods for buying them. What was his commission? What was the total cost of the goods to the merchant?

3. A friend of mine, who was an automobile salesman, was going to a distant city. I asked him to buy for me a certain used car if he could find one. He bought one for me and paid $1175 for it. If he charged 5% for buying, how much was his commission?

4. A brokerage agent bought for me $2575 worth of stocks. He charged me $\frac{3}{5}$% commission. How much was his commission?

5. A certain large summer hotel sent a man through the country buying eggs. If he bought 1800 dozen eggs at an average price of 26 cents a dozen and charged 3% commission for buying them, how much did he earn? How much did the eggs cost the hotel per dozen?

6. A commission man bought 600,000 pounds of lard for another man at $3.90 per 100 pounds. He charged $\frac{2}{5}$% commission. What was his commission? What was the total cost of the lard including the commission?

7. Find the commission and gross cost of each of the following:

BUYING PRICE	RATE OF COMMISSION	COMMISSION	TOTAL COST
$90	2%		
$1200	$2\frac{1}{2}$%		
$800	$4\frac{1}{2}$%		
$524.50	6%		
$360.40	$3\frac{1}{8}$%		

8. A book agent sold 60 sets of illustrated lectures at $40 a set. He received a commission of 25%. How much money did he receive? How much did he turn in to the publisher?

9. Obtain any problem in commission from your parents or from any business man that you know and bring it to class with you.

More Algebra

Word problems, using words and letters in place of numbers in mathematics.

1. How do you find the product of any two numbers? What is the product of two numbers?

If the multiplicand were M and the multiplier were m, what would you do with them to find the product?

2. If you know the total cost of running a state for one year and the number of people in that state, how could you find the cost per person? If c represented the cost and p represented the number of people, $c \div p = ?$

3. If you knew the average number of pounds per cow of butter fat produced by a herd of cows for a month and the number of cows in that herd, how could you find the total number of pounds of butter fat produced by that herd in that month?

What letters could you use to stand for the things mentioned above?

4. If you knew the thickness of one page of a book, what would you have to know to find the thickness of the book exclusive of covers?

5. If you knew how much a boy earned in a year and how much he spent, how could you find what he saved?

6. If you knew the value of a franc in United States money, how would you find the number of francs in a certain number of dollars?

7. How would you find which was the cheaper if you knew the price per gallon and the price per pint?

8. What would you have to know and how would you proceed to make a graph of all of the standings of your class in every subject?

Review

1. What is the rate of gain on the cost if goods are bought for \$12.50 less 20%, and sold for \$12.50?

2. A merchant bought a certain article for \$5.00 and marked it so as to gain 20% on the cost, but on account of poor sales he had to reduce his marked price $33\frac{1}{3}$%. What was his new marked price?

3. $\frac{3}{4}$% of \$4216 = ?

4. Mr. Brown owes \$3060. This is $37\frac{1}{2}$% of what he is worth. How much is he worth?

5. If $2\frac{1}{2}$ pounds of meat cost \$.95, how many pounds of that kind of meat will \$3.61 buy?

6. A certain number increased by $\frac{4}{15}$ of itself equals $3\frac{4}{5}$. What is the number?

7. 1200 bushels of wheat are sold for \$1080. If the wheat was sold at a loss of \$20, what was the cost per bushel?

8. If a real estate agent received $\frac{1}{20}$ of the selling price for selling a house and he received \$475 as his pay, for how much did the house sell?

9. A shoe merchant sold a pair of shoes for \$8.00. If he made 15% of the selling price, what was his margin of profit?

10. \$1000 is $\frac{1}{6}$ less than what?

11. If you knew the selling price and the loss, how could you find what part the loss is of the selling price? Of the cost?

12. $1\frac{2}{3}$% of \$2400 = ?

13. In a certain architect's drawing, one room is marked 11′ 6″ (11 feet 6 inches) wide. The drawing measures $1\frac{3}{4}$ inches. How many feet will 1 inch represent on that drawing?

14. A merchant planned to make 10% on the total cost of some goods. If the goods cost him $4750.10, and freight and overhead was $480, for how much must he sell the goods?

15. A merchant sold goods one year that cost him $175,675 at an average gross gain of 27% of the first cost. His overhead expenses were $26,876. Find his gross profit. Find his net margin of gain after deducting his overhead expenses.

Aliquot Part Drill

Samples. $24 \times .125 = 24 \times \frac{1}{8} = 3.$
$32 \times 12\frac{1}{2} = 32 \times \frac{1}{8} \times 100 = 400.$

1. $16 \times .75$
2. $32 \times .375$
3. $60 \times .16\frac{2}{3}$
4. $128 \times .25$
5. $80 \times .875$
6. $720 \times .11\frac{1}{9}$
7. $40 \times .20$
8. $3.6 \times .33\frac{1}{3}$
9. $6.4 \times .125$
10. $1600 \times .06\frac{1}{4}$
11. $40 \times 12\frac{1}{2}$
12. $72 \times 16\frac{2}{3}$
13. $18 \times 33\frac{1}{3}$
14. 64×25
15. $80 \times 6\frac{1}{4}$
16. $72 \times .75$
17. $400 \times .62\frac{1}{2}$
18. $9.6 \times .37\frac{1}{2}$
19. $270 \times .11\frac{1}{9}$
20. $350 \times .14\frac{2}{7}$

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER IX

In Chapter IX you have been doing the work so common to business men and to all of us who without exception deal with them. You must begin to realize how important it is that you be able to use intelligently and easily the simple business methods involved in the per cent of loss or gain, rate of loss or gain, selling price, overhead, margin, net profit, discounts, successive discounts, rate of discount, marked price, etc.

Test your skill in using some of these business methods by taking again the four short I. R. B. tests of this chapter. Remember: independent work, improved scores, the I. R. B. entries, and your 100% record.

CHAPTER X

THE FOUNDATION OF BANKING BUSINESS. INTEREST

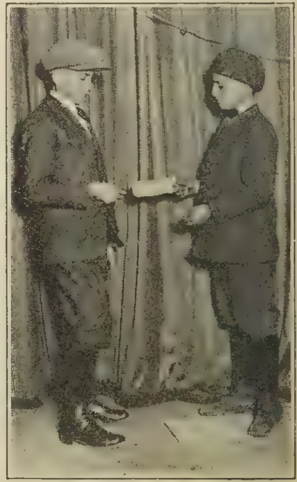
Interest. — In the first three chapters of our business work, beginning with Part II of this book, we studied

A GREAT BANKING CENTER — WALL STREET

decimals so as to master percentage and then we applied percentage to a number of business problems. We are now to take up another application of percentage used by everybody who lends or borrows money, or who puts money in the bank.

Here is Joe borrowing a dollar of Alec. In the next picture Joe is paying Alec back the dollar, with five cents extra for the use of it. This five cents is the *interest*. Interest is money paid for the use of money.

Borrowing Money



Paying Interest

If we live in a house belonging to somebody else, we have to pay rent for the use of it. We pay for the use of an automobile when we hire it at a garage. Similarly, it is right for us to pay interest when we use another person's money. The borrowed money is called the *principal*. The sum of the principal and interest is called the *amount*.

Interest is only another form of percentage. It is usually reckoned at so much per cent a year. The duration of the loan is called the *time*. The per cent of the principal which is charged is called the *rate*.

The standard rate is 6% a year. In solving interest problems a year is reckoned as 360 days and each month as 30 days.

The Simplest Way to Compute Interest. — When interest is 6% a year, for 2 months it is $\frac{1}{6}$ of 6% or 1%. We have already learned that 1% of a number is found simply by pointing off two decimal places to the left. Thus interest on \$125.00 for two months would be \$1.25.

If the time is part of a year, reduce it to a multiple or fraction of two months, and multiply this by the 1% of the principal.

Illustrative Example 1. Find the interest on \$375.75 for four months at 6% a year.

Solution: Move the decimal point two places to the left to get the interest for two months at 6%. This gives \$3.7575. Multiply the two months' interest by two for the four months' interest, and we obtain $\$7.51\frac{1}{2}$, or \$7.52. *Answer.*

Illustrative Example 2. Find the interest on \$420.00 for 2 years, 7 months, and 15 days at 6%.

Solution:

2 × 6% of \$420 = \$50.40 interest for 2 years.
3 × \$4.20 = 12.60 interest for 6 months.
(\$4.20 = 2 months' interest)
$\frac{3}{4}$ × \$4.20 = 3.15 interest for 1 mo. 15 da.
($\frac{3}{4}$ of 2 months' interest)
\$66.15 *Answer.*

Finding Interest in Your Work

Find the interest at 6% on the following principals for the time given:

1. $500 for 2 months.
2. $1200 for 6 months.
3. $840 for 1 year and 4 months.
4. $1500 for 2 months and 15 days.
5. $9000 for 20 days.
6. $1250 for 2 years and 10 months.
7. $16,800 for 3 years, 6 months, and 15 days.
8. $20,000 for 9 months and 15 days.
9. $3000 for 5 years, 1 month, and 15 days.
10. $750 for 3 months and 15 days.

Variation of the 6% Method. — While the method just studied works best for interest at 6%, it may be used also for other per cents.

$.06 = the interest on $1 for 1 year at 6%.
$.005 = the interest on $1 for 1 month at 6%.
$.000⅙ = the interest on $1 for 1 day at 6%.

Note. In finding interest count a month as 30 days.

Illustrative Example. Find the interest on $1500 for 2 yr. 7 mo. 12 da. at 5%.

Solution:

First we find the interest on $1.00 for the given time.

$.12 = int. on $1 for 2 yr. at 6%.
.035 = int. on $1 for 7 mo. at 6%.
.002 = int. on $1 for 12 da. at 6%.
$.157 = int. on $1 for 2 yr. 7 mo. 12 da. at 6%.

Then we multiply this by the principal to get the total interest at 6%.

$.157
1500
$235.50 = int. on $1500 for 2 yr. 7 mo. 12 da. at 6%.

Next we divide this by 6 to get the interest at 1%.

6)$235.50
39.25 = int. on $1500 for 2 yr. 7 mo. 12 da. at 1%.

Finally we multiply the 1% by 5 to get the answer.

39.25
× 5
$196.25 = int. on $1500 for 2 yr. 7 mo. 12 da. at 5%.

Or we may simply subtract the 1% interest from the 6% interest. This will give us the answer, 5% interest.

6)$235.50
– 39.25
$196.25 = *Ans.*

UNANIMOUS 100% CLASS RECORD BY THE C. C. AND C. C. PLAN

In the 10 examples, page 252, work for a 100% record by every pupil in the class. Coöperation on the C. C. and C. C. plan will accomplish this. As soon as a pupil secures his own 100% he should be appointed by the teacher and the Coaching Committee as a coach for some other pupil who is experiencing difficulty. These appointments should be made until the class is evenly divided into Coach and Pupil Couples.

The Coaching Committee should supervise the coaches to make sure that they assist the pupils only when necessary. It may not be possible in class-time for some pupils to earn this score; let coaches volunteer to assist these pupils in out-of-class-time. Then on the following day the 100% record may be made unanimous for the class.

Rapid Calculation Mentally

1. What is the interest on $500 for 1 year at 6%? on $700? on $3000?

2. Find the interest on $400 for 1 month at 6%; on $60; on $1000.

3. What is the interest on $600 for 1 day at 6%? on $1200? on $6000?

4. If the interest for 1 year is $12, what will it be for 3 years, 4 months?

Taking an Interest

1. Find the interest on $360 for 1 year, 6 months, 12 days at 6%.

2. What is the interest on $840 for 2 years, 8 months, 24 days at 6%?

3. Find the interest on $6000 for 4 months at 6%.

4. What is the interest on $120,000 for 12 days at 6%?

5. A man borrowed $1800 at the bank for 1 month, 15 days at 6%. How much interest should he have to pay the bank?

6. Mr. Wood borrowed $18,000 at 6%. On account of a poor wheat crop he was unable to pay this back until 1 year, 4 months, and 15 days afterwards. How much should he pay back, including interest?

7. A manufacturer borrowed $50,000 at 6% to carry on an advertising campaign. On account of increased business and gains he was able to pay this back after 9 months, 18 days. How much should he pay back?

8. The principal is $2400, the time is 4 years, 9 months, 6 days, and the rate is 6%. What is the interest?

9. Find the interest on $4000 for 5 years, 3 months, 28 days at 5%.

10. What is the interest on $1500 for 1 year, 4 months, 24 days at 4%?

The General Method. — As interest is a form of percentage, we may work interest problems just as we do percentage problems. This is very simple when the time is an integral number of years, or when it is an aliquot part (see page 36) of a year.

Illustrative Example 1. Find the interest on $1850 for 2 years, 3 months at 5%.

Solution: $1850
.05
$92.50 = interest for 1 year at 5%.
2
$185.00 = interest for 2 years at 5%.
23.125 = interest for 3 months at 5%.
$208.125 = interest for 2 years, 3 months at 5%.

Explanation:
4)$92.50 = 1 year's interest at 5%.
$23.125 = 3 months' interest, because 3 months is $\frac{1}{4}$ of 1 year.

Illustrative Example 2. Find the interest on $625 for 4 years, 2 months, 10 days at 4%.

Solution:
$625
.04
$25.00 = 1 year's interest at 4%.
4
$100.00 = 4 years' interest at 4%.
4.166 = 2 months' interest at 4%.
2 mo. = $\frac{1}{6}$ of 1 year.
.694 = 10 days' interest at 4%.
10 da. = $\frac{1}{6}$ of 2 months.
$104.86 = interest for 4 years, 2 months, 10 days at 4%.

Note. Remember that in finding interest, a business month is considered as having 30 days.

Illustrative Example 3. Find the interest on $1800 for 3 years, 6 months, 16 days at 3%.

Solution:

$1800
.03
$54.00 = int. for 1 year at 3%.
$162.00 = “ “ 3 years at 3%. (6 mo. = 4 mo. + 2 mo.)
18.00 = “ “ 4 months at 3%. 4 mo. = $\frac{1}{3}$ of 1 yr.
9.00 = “ “ 2 “ “ 3%. 2 mo. = $\frac{1}{2}$ of 4 mo.
2.25 = “ “ 15 days “ 3%. 15 da. = $\frac{1}{4}$ of 2 mo.
.15 = “ “ 1 da. “ 3%. 1 da. = $\frac{1}{15}$ of 15 da.
$191.40 = “ “ 3 yr. 6 mo. 16 da. at 3%.

Note. The months should be divided up so that either 1 month's or 2 months' interest appears in the work in order that the number of days may be broken up so that they are an aliquot part of either 1 or 2 months.

This leads to the following rule:

1. *Multiply the principal by the rate of interest. Multiply this result by the number of years if there is more than one year.*
2. *Change the number of months into aliquot parts of one year. By the use of these aliquot parts find the interest for the number of months required.*
3. *Change the number of days (if necessary) into aliquot parts of 30 days or of 60 days.*
4. *Find the interest for the required number of days.*
5. *Find the total interest.*

Do-It-in-Your-Head Practice

1. What is the interest on $100 for 1 year at 6%? for 2 years? for 5 years?

2. 6 months is what part of 1 year?

3. How much would be the interest on $100 for 6 months at 6%?

4. What part of a year is 3 months? 4 months? 1 month?

5. What part of 1 month is 5 days? 6 days? 15 days? 10 days? 2 days? 1 day?

I. R. B. Test [**1**]. A Standard Time (for your class) to do the work of this and the other tests of this chapter should be fixed by your Steering Committee and Teacher (ex officio) when you take the First Trials. Prepare the I. R. B. Chart for Chapter X (7 tests). Enter your First Trial scores for [1].

Do-It-in-Your-Head Plus Paper-and-Pencil Practice in Interest

Solve the following ten problems.

1. Find the interest on $840 for 5 years at 6%.

2. What is the interest on $1020 for 3 years at 5%?

3. Find the interest on $320.50 for 2 years at 8%.

4. What is the interest on $1650 for 2 years, 6 months at 6%?

5. Find the interest on $1280 for 3 years, 4 months at 6%. What is the amount? See page 248.

6. What is the interest on $150 for 6 years, 2 months at 6%?

7. Find the interest on $2530 for 3 years, 8 months at 5%. What is the amount?

8. What is the interest on $1600 for 4 years, 9 months, 15 days at 6%?

9. Find the interest on $3600 for 2 years, 5 months, 16 days at 5%.

10. Principal $4800, time 3 years, 10 months, 23 days. Rate 6%. Find the interest.

Finding the Time between Two Dates. — To find the time between two dates we subtract the *earlier* date from the *later*. In order to do this we must know the *number* of each month as shown in the following table at the right.

1. January
2. February
3. March
4. April
5. May
6. June
7. July
8. August
9. September
10. October
11. November
12. December

Illustrative Example 1. — Find the time between March 10, 1926, and August 16, 1927.

Solution: March is the 3d month and August is the 8th month.

	YEAR	MONTH	DAY
We write	1927	8	16
	1926	3	10
Subtracting	1	5	6

Hence the time is 1 year, 5 months, 6 days.

Illustrative Example 2. Find the time between November 12, 1921, and September 6, 1927.

Solution:

YEAR	MONTH	DAY
1927	9	6
1921	11	12
5	9	24

Explanation: We cannot take 12 days from 6 days, so we borrow a whole month from 9, leaving 8. Thus we have 36 days from which to take 12 days. This leaves 24 days.

We cannot take 11 months from 8 months, so we borrow a whole year, leaving 1926. The year that we borrowed consists of 12 months. Thus we have 20 months from which to take 11 months, leaving 9 months. Subtracting 1921 from 1926 we have 5 years. Hence the time is 5 years, 9 months, 24 days.

Illustrative Example 3. Find the exact number of days from June 12, 1927 to September 21, 1927.

Solution:

18 days left in June
31 days in July
31 days in August
21 days in September
101 days from June 12 to September 21

Oral Test

1. How many actual days from March 5 to March 20?

2. How many actual days from June 20 to July 10?

3. What is the exact number of days from September 15 to October 20?

4. Find the number of actual days from February 12 to March 15, in a leap year.

I. R. B. Test [2]. Enter First Trial scores. In this exercise ask no help of any one. Work the 16 examples by yourself in any way you choose until you have a 100% record. Stick to it even if you are obliged to work overtime. Remember " A winner never quits and a quitter never wins."

Find the time between the following dates:

1. August 8, 1923 and November 18, 1927.

2. March 5, 1921 and October 15, 1927.

3. From Peary's discovery of the pole April 6, 1909 to Byrd's flight over the pole May 9, 1926.

4. From the signing of the Declaration of Independence July 4, 1776, to the " Sesqui " celebration of it in Philadelphia July 4, 1926.

5. From the beginning of regular air mail service between New York and San Francisco, July 1, 1924, to today.

6. From the beginning of the World War, August 3, 1914, to the end, November 11, 1918.

7. From the battle of Bull Run, July 21, 1861, to the battle of Gettysburg, July 1, 1863.

8. From the start of the round-the-world flight, March 17, 1924, to its end, September 9, 1924.

THE AROUND-THE-WORLD FLIERS ARRIVING OVER NEW YORK CITY

9. Abraham Lincoln was born Feb. 12, 1809, and died April 15, 1865. How long did he live?

10. Find the interest on $440 from January 10, 1923 to August 20, 1927 at 6%.

11. What is the interest on $680 for the exact number of days from June 6 to August 5 at 6%?

12. Find the interest on $1820 from September 3, 1924 to November 18, 1927 at 5%.

13. How much interest will pay for a loan of $1200 from October 5 to November 4 at 6%? How much will pay the whole indebtedness? (Amount = prin. + int.)

14. Mr. Henry borrowed $12,000 on March 6 and repaid it on May 5 at 6% interest. How much did he owe at that time?

15. \$625 was borrowed May 4, 1922 and repaid with interest on September 10, 1927. What amount was due at 6% interest?

16. The Battle of Concord Bridge occurred April 19, 1775, and the firing on Fort Sumter began January 9, 1861. What was the length of time between these dates?

CONCORD BRIDGE

The famous statue of the Concord Minute-Man can be seen at the farther end.

How a Formula of Algebra Helps to Compute Interest. — We have had enough examples to find out that the process for working interest examples is always the same. Whatever short-cuts we may use, we always multiply the principal by the rate and this result by the time. This may be expressed by the formula

$$\text{Interest} = \text{principal} \times \text{rate} \times \text{time (in years)}$$

Or more briefly $\mathbf{I = prt.}$

Illustrative Example 1. We wish to use this formula to find the interest on \$640 for 4 years at 6%.

Solution:

$$\begin{array}{rcl} I & = & p \qquad r \qquad t \\ \text{Interest} & = & \$640 \times .06 \times 4 \\ & = & \$153.60 \end{array}$$

Illustrative Example 2. Find by the formula, the interest on \$1800 for 2 years 4 months at 4%.

Solution:

$$\begin{array}{rcl} I & = & \$1800 \times .04 \times 2\tfrac{1}{3} \\ & = & \$168 \end{array}$$

Illustrative Example 3. Find the interest on \$720 for 3 years, 2 months, 10 days at 5%.

Solution:

$$\begin{array}{rcl} 3 \text{ years} & = & 3 \text{ years} \\ 2 \text{ months} & = & \tfrac{1}{6} \text{ of a year} \\ 10 \text{ days} & = & \tfrac{10}{360} \text{ or } \tfrac{1}{36} \text{ of a year} \end{array}$$

$$\text{Total time} = 3 + \tfrac{1}{6} + \tfrac{1}{36} = \frac{108 + 6 + 1}{36} = \tfrac{115}{36} \text{ years}$$

Note. Count 360 days to the business man's year unless otherwise specified.

Now

$$I = \frac{\overset{36}{\cancel{720}}}{1} \times \frac{\overset{1}{\cancel{5}}}{\underset{20}{\cancel{100}}} \times \frac{115}{\cancel{36}} = \$115$$

I. R. B. Test [3]. Enter First Trial scores. Try again to make your own 100% record. Using the formula from algebra, find the interest on each of the following:

1. \$630 for 5 years, 4 months, at 6%.
2. \$1175 for 2 years, 10 months, at 4%.
3. \$3675 for 1 year, 7 months, 15 days, at 6%.
4. \$560 for 3 years, 4 months, 8 days, at 7%.
5. \$1625.50 for 1 year, 6 months, 12 days, at 6%.
6. \$1305 on interest from August 1, 1921 to September 10, 1927, at 6%.

7. $560 for 90 days, at 6%.

8. $750 for 4 months, at 6%.

9. $2400 for 6 months, 15 days, at 6%.

10. I borrowed $860 on April 8, 1926 and repaid it June 20, 1927 with interest at 6%. What amount should I repay the man from whom I borrowed the money?

Review for Rapid Calculators

1. First find the percentage, then find the total of the percentage column.

	PERCENTAGE		PERCENTAGE
4% of $500 =		12.5% of $320 =	
2% of $800 =		37.5% of $720 =	
5% of $700 =		87.5% of $2400 =	
8% of $400 =		62.5% of $4000 =	
9% of $1200 =		4.5% of $800 =	
½% of $16,000 =		2.25% of $1600 =	

2. The St. Louis team of the National League won the

THE ST. LOUIS CARDINALS
World's Champions in 1926.

pennant in 1926. They won 89 games out of the 154 played. What per cent did they win?

3. A real estate agent received 3% for selling a house. If he received \$240 as his pay, for what did he sell the house? How much did the seller receive?

4. Draw a line on the board 100 inches long. Now cut off 25% of it. 45%. 65%. 84%. 97%. $12\frac{1}{2}$%. $\frac{3}{4}$%.

5. (*a*) Five times 6 is 2 times what number?

(*b*) $\frac{1}{2}$ of $\frac{1}{6}$ is $\frac{1}{12}$ of what number?

THE FOUR FUNDAMENTAL PROCESSES BY TEAM WORK

Steering Committee and Teacher appoint four team captains; these in turn choose teams. Steering Committee set standard time for Parts 2, 3, and 4 which with Standard time for Part 1 should be entered in each I. R. B.

All teams work each part at the same time; write answers only. When all teams have completed Part 1 begin on Part 2 and so on; write all answers on one sheet. Each pupil keep record of his own time for each part.

When Part 4 is finished captains collect papers, exchange, and distribute to teams. Chairman of Steering Committee read correct answers while team members check and score papers; Steering Committee explain how to score each Part. Each pupil preserve his own accuracy score. Captains average team scores for teams whose papers were received in exchange; average score for each Part.

Steering Committee draw on blackboard graphs of the average scores of the four teams. These graphs will show the winning team of each part. Can you also find from the graphs in which of the four fundamental processes your class needs continued drill?

REVIEW TRIAL OF I. R. B. TESTS, CHAPTER IX

There were four short I. R. B. tests in Chapter IX. Look over the pages of the preceding chapter to locate them and examine in your I. R. B. the time and accuracy scores for the First and End-of-Chapter records. Now take these tests again. Stick-To-It until you improve the scores and then enter them in the Review Trial column.

I. R. B. Tests [4]. Enter time and accuracy scores for Tests [**4**], [**5**], [**6**], and [**7**] in the First Trial column.

PART 1

Practice in Addition

[**4**]. See how many you can do correctly in 3 minutes.

$65.75	$ 7.16	$12.04	$ 4.87	$20.32
8.12	15.28	36.17	19.07	11.12
9.06	7.70	7.25	23.16	2.85
15.16	8.40	12.27	5.26	4.48
7.28	17.06	6.37	2.18	56.42
8.32	7.06	7.87	6.37	8.32
32.24	72.54	42.18	47.08	5.51

PART 2

Practice in Multiplication

[**5**]. See how quickly you can do the following. Have the Steering Committee set a standard time.

124	561	624	824	400	1600
$12\frac{1}{2}$	$9\frac{1}{3}$	$7\frac{1}{4}$	$5\frac{1}{8}$	$3\frac{3}{4}$	$3\frac{1}{16}$
450	390	525	960	575	1200
$4\frac{4}{9}$	$6\frac{2}{3}$	.01	.04	.2	$.00\frac{1}{2}$

125% of $400. 150% of $600. 250% of $800. 375% of $1200.
$2\frac{1}{2}$% of $500. $3\frac{3}{4}$% of $1600. $5\frac{1}{4}$% of $2000. $6\frac{2}{3}$% of $2400.

PART 3

Practice in Subtraction

[**6**]. Have the Steering Committee set a time.

62.5	561.04	24.016	321.2316	23.0004
9.7	96.16	9.257	236.7439	22.0875

$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$	$3\frac{1}{2}$	$15\frac{1}{2}$	$296.5\frac{1}{2}$	$568\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$2\frac{1}{4}$	$13\frac{3}{4}$	$157.4\frac{1}{4}$	356.125

PART 4

Practice in Division

[**7**]. Steering Committee set time.

$12\overline{)132}$ $9\overline{)234}$ $8\overline{)344}$ $10\overline{)567.32}$ $1000\overline{)456.23}$ $20\overline{)46.80}$

$30\overline{)6090}$ $60\overline{)3204}$ $125\overline{)2000}$ $250\overline{)80{,}000}$ $2.5\overline{)12.5}$ $6.5\overline{)19.5}$

Word Problems

Wherever practicable write your answer as a formula. For instance in example 1,

Interest = principal × rate × time in years.

Then simplify by substituting the letters. *E.g.*, **I = prt.**

1. Given the principal, rate, and time in years, how do you find the interest?

2. If you knew the interest on a certain sum of money for one year, how would you find it for two years? for two and one-half years? for six months?

3. If you had a sum of money left you, where would you go to find out a good place to invest it?

4. If you knew the margin of profit in money and the rate per cent of margin on the cost, how could you find the cost?

5. If you knew the amount that a real estate agent had received for collecting some money and also knew the rate per cent that he charged for collecting, how would you find the amount of money that he collected?

6. If you knew the total attendance in a school for a school year of 200 days, how could you find the average attendance per day?

7. If I know the selling price and the loss in money, how shall I find the cost?

8. If Mr. Gladstone knew how much his property was insured for and also knew the rate per cent of insurance that the company charged for insuring it, how would he find the cost of his insurance?

9. If Henry knew how much he had earned each day during his summer vacation, how could he find his average daily earnings?

10. If George knew that Henry had averaged a certain amount each day, and he knew exactly how many days Henry had worked, how would he find out the amount that Henry had earned?

Systematic Review of Interest

Find the interest on

1. $1500 at 6% for 3 years.
2. $2000 at 5% for $2\frac{1}{2}$ years.
3. $1600 at $3\frac{1}{2}$% for 2 years.
4. $525 at 4% for 2 yr. 4 mo.
5. $200 at 6% for 60 days.
6. $900 at 6% for 20 days.
7. $30,000 at 6% for 2 days.
8. $120,000 at 6% for 1 day.
9. $6000 at 6% for 45 days.
10. $9000 at 6% for 120 days.
11. $7200 at 6% for 12 days.
12. $12,000 at 6% for 6 days.
13. $575.75 for 60 days at 6%.
14. $824.50 for 15 days at 6%.
15. $75 for 60 days at 6%.

Investing Our Savings. — When you have saved some money, you naturally wish to invest it so that it will bring the largest possible safe interest. It is always wise to ask your banker before you invest money. The following examples refer to safe investments.

Some Wise Investors

1. Mr. Jones invested $5200 in a mortgage on a house which paid him 6% interest. What annual income did he receive for it?

2. Mr. Alvord spent $6000 in buying U. S. bonds at par. (At par means $1000 in this case.) These pay him 4% interest. What annual income did he receive from this amount of bonds?

3. Mr. Taylor has $3500 deposited in a bank which pays him 3½% interest. How much annual interest does his money earn for him?

4. A business man borrowed $12,000 in New York at 6% and loaned $8000 of it to a man in Idaho at 8% and $4000 to a man in Nevada at 9%. How much would he make each year?

5. Henry earned and saved up during the summer $75. He put it in the local town bank and received 4% interest on it. How much would he have to his credit in that bank at the end of one year?

6. Mr. Williams loaned $3400 to Mr. Stewart. If Mr. Williams received this money back with interest at 6% at the end of 90 days, how much interest had Mr. Williams' money earned for him?

7. Mr. Williams again loaned out $1500 and received it back at the end of 35 days with interest at 6½% per annum. How much interest did he receive this time for the use of the money that he had loaned?

8. A bank in New York loaned a very rich man $100,000 for 12 days at 6% per annum. How much interest should the bank receive?

9. John Burgin deposited $75 in a bank on his sixteenth birthday. The bank paid 4% interest a year. If he left his original sum in that bank until his twenty-first birthday, how much interest would it earn?

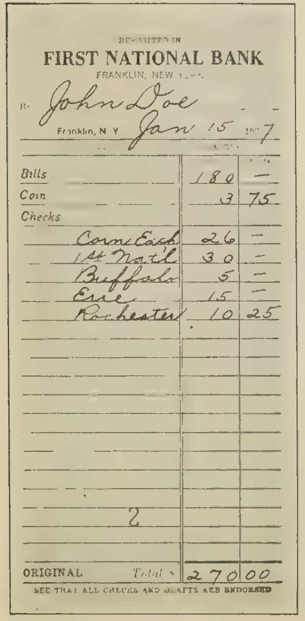
10. Mr. Baker spent $8000 for Liberty Bonds which paid $4\frac{1}{4}\%$ a year. What did he receive each year?

Banks. — We have just spoken of saving and investing. Both are helped by banks. Banks are the most important institutions in the business world. Without them modern business could not exist. So we must learn about them.

THE INTERIOR OF A BANK

Many towns have school banks where the pupils may deposit their savings, but whether banks are large or small they all have certain things in common.

Opening an Account. – John Doe earned $270 and decided to put it in the bank, that is, to *open an account.* He filled out a deposit slip and gave it with his money and checks to the *receiving teller*, who made the proper entry in a *bank book*, which he gave John. In future, when John makes a deposit, he hands the teller his money, deposit slip, and bank book. The teller returns the bank book after recording each deposit.

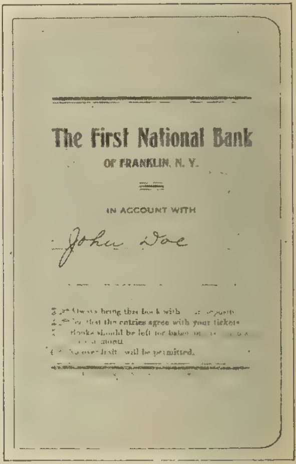


DEPOSITED IN
FIRST NATIONAL BANK
FRANKLIN, NEW YORK
By John Doe
Franklin, N. Y. Jan 15 1907

Bills	180	—
Coin	3	75
Checks		
Corn Exch	26	—
1st Nat'l	30	—
Buffalo	5	—
Erie	15	—
Rochester	10	25
ORIGINAL Total	270	00

SEE THAT ALL CHECKS AND DRAFTS ARE ENDORSED

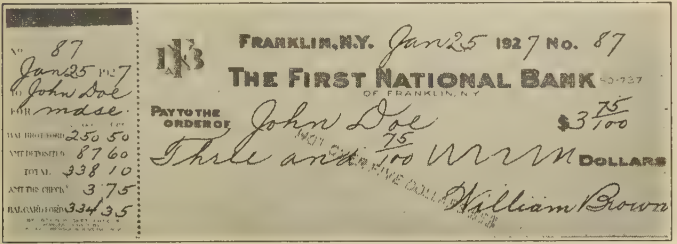
Deposit Slip



The First National Bank
OF FRANKLIN, N. Y.
IN ACCOUNT WITH
John Doe

Bank Book

Drawing Checks. — To provide for payments from an account, the bank furnishes a *check book* containing *checks* and *stubs.* A properly drawn check is an order for the bank to pay a specific sum to the person named. On the stub may be recorded the facts of each check drawn as well as the changes in the *balance*. The bank balance is the amount of money actually on deposit. In order to keep this record up to date, each check paid out should be subtracted and each deposit should be added. Here is a sample check with its stub.



[redacted]

No. 87
Jan 25 1927
To John Doe
For mdse.
Bal. Bro't Forw'd 250 50
Am't Deposited 87 60
Total 338 10
Am't this Check 3 75
Bal. Car'd Forw'd 334 35

FRANKLIN, N.Y. Jan 25 1927 No. 87
THE FIRST NATIONAL BANK 50-737
OF FRANKLIN, N.Y.
PAY TO THE ORDER OF John Doe $3 75/100
Three and 75/100 DOLLARS
William Brown

STUB CHECK

If Mr. Brown wishes to draw some money for his own use he should make the check payable to himself or to *Cash*.

Indorsing Checks. — Mr. Grogan owes a sum of money to A. S. Sears Coal Co. He received a check from John Doe which he wishes to apply on this account. So he writes on the back of the check his indorsement like the one at the right. This is called a *complete indorsement* or an *indorsement in full*. If he had simply written his name like the indorsement at the right, the check would have been payable to *bearer*, that is, to anybody who presented it to the *paying teller* at the bank. This is called a *blank indorsement*.

Pay to
A. S. Sears Coal Co.
Samuel Grogan

Samuel Grogan

If Mr. Grogan deposits the check to his own account, he indorses it in blank.

If you are mailing your bank a check, which is to be deposited to your account, you should indorse it like this.

> *For deposit only*
> *in the Treadwell*
> *First National Bank*
> *John Doe*

The object of this is to prevent anybody else from having it *cashed.*

Checks are always indorsed across the back of the *left* end.

Banking

1. Explain why it is safer to pay bills by check than to send the money itself.

2. Ask your father or your teacher to explain to you how a canceled check serves as a receipt.

3. Will Mr. Grogan's check be cashed by any other bank than the one on which it is drawn? Explain.

4. Suppose you have money in the First National Bank of Sacramento, Cal. Draw a check on this bank for $37.12, payable to Samuel Tyler.

5. Indorse this check in full to Sears, Roebuck and Co.

6. John Doe owes Henry Davie $25.75. Mr. Doe has money in the Second National Bank of Delhi. Write the check to pay this bill in full.

7. George Williams owes Charles Scott $150. He has been owing this bill for 2 years and 6 months. Mr. Scott sends him a bill for the amount plus the interest at 6%, at the end of this time. Mr. Williams now has the money in the Third National Bank of Unadilla to pay the bill, so he computes the interest and sends a check for the full amount. Write this check.

8. Norman Jackson bought an automobile of Johnson Bros. on May 1. He paid cash all except $400. He gave four notes as follows for the balance: $100 due June 1, $100 due July 1, $100 due Aug. 1, and $100 due September 1. Each note was dated May 1. Mr. Jackson paid each note and the interest on it at 6% when the note became due by checks on the Third National Bank of Fayetteville, N. Y. Find the amount due on each note when each note was due, then write each check.

I. R. B. RECORD OF THE ALGEBRAIC FORMULAS OF CHAPTER X

In this chapter there is one very important formula, that for interest, on page 259. Place this among your records in your I. R. B. You will recall that you wrote your answers to some of the "Word Problems" on page 264 in formulas. Enter as many as you think worth preserving with the interest formula.

FOURTH TRIAL OF I. R. B. TESTS OF CHAPTERS VI AND VII

Unless you have planned your own Fourth Trial Tests to complete the I. R. B. Charts, you will find this a good opportunity to take the Fourth Trial of the I. R. B. tests of Chapters VI and VII. If you do not succeed in improving the scores of the first three trials, read again the directions for Fourth Trial in Chapter V.

More Clinching of a Good Habit

1. If a man had a balance of $125.75 in the bank and deposited $250, then gave out checks of $36.40, $8.75, $19.43, $52.35, and $7.50, how much balance would he then have in the bank?

2. A merchant had a balance of $350.75 in the bank on the morning of Dec. 5. He deposited during that day checks and money as follows: $23.50, $50, $15, $18.60, and $40.75. He drew out checks during that day of $28.50,

$49.16, $83.50, $125, and $75.42. How much balance had he left in the bank that night?

3. George Ham, a merchant, had a balance in the bank of $275.56. He deposited checks and money as follows: $45.50, $53.16, $2.35, $4.50, and $75. If he drew out checks of $43.16, $5.67, $18.45, and $24.80, what balance had he left in the bank?

4. You deposit in your bank $15 in coins, $34 in bills, and a check on the First National Bank made by John Doe for $60. Make out the deposit slip.

5. You receive a bill from Brown Bros. of $245.50 less discounts of 20% and 10%. Find the net amount of this bill. Write the check to pay it.

How Payments Are Made by Mail. — Cash should not be sent by mail because it may be stolen or lost. Loose silver is likely to wear a hole through the envelope. If the sender has a bank account, money may best be sent by check, as explained on page 268.

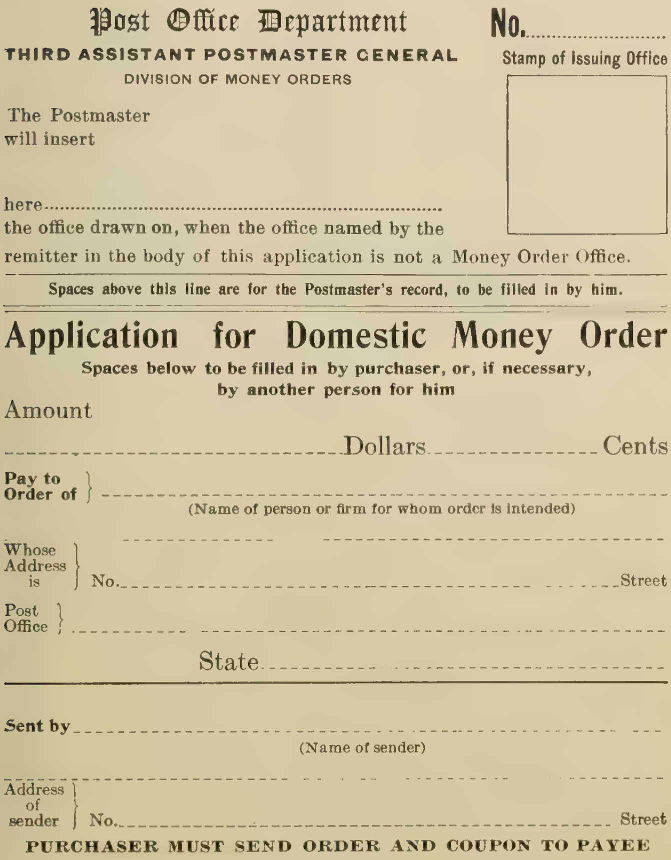
Other methods of paying bills by mail are by

Postal money order
Express money order
Bank drafts
Telegraphic money order.

Postal money orders are very handy for paying small bills at a distance. The sender fills out a blank like that on page 273.

The sender then pays his home post office the amount of the money to be sent plus the fee for the money order. This money order is sent to the one to whom the bill is to be paid. The receiver takes it to his own post office when he gets it and there receives the full amount of the money order.

Sample Money Order Application Blank



Post Office Department No.............................

THIRD ASSISTANT POSTMASTER GENERAL

DIVISION OF MONEY ORDERS

Stamp of Issuing Office

The Postmaster
will insert

here..
the office drawn on, when the office named by the remitter in the body of this application is not a Money Order Office.

Spaces above this line are for the Postmaster's record, to be filled in by him.

Application for Domestic Money Order

Spaces below to be filled in by purchaser, or, if necessary, by another person for him

Amount

______________________________Dollars______________Cents

Pay to Order of ____
(Name of person or firm for whom order is intended)

Whose Address is No.____Street

Post Office ____

State____________________________________

Sent by ____
(Name of sender)

Address of sender No.____Street

PURCHASER MUST SEND ORDER AND COUPON TO PAYEE

He may, if he prefers, indorse the money order over to his own bank if he has a bank account.

The express and telegraph companies issue money orders much as the post office does. The money order of a bank is called a *draft*.

The following are the rates now required by the U. S. Post Office Department for money orders.

For Orders

From $0.01 to	$2.50	5 cents
From $2.51 to	$5.00	7 cents
From $5.01 to	$10.00	10 cents
From $10.01 to	$20.00	12 cents
From $20.01 to	$40.00	15 cents
From $40.01 to	$60.00	18 cents
From $60.01 to	$80.00	20 cents
From $80.01 to	$100.00	22 cents

Paying by Mail

1. How much would you have to pay as a fee to send each of the following by money order: $2.75? $14.50? $.27? $40.02? $39.99?

2. How much would you have to pay into the post office in order to send each of the above by money order?

3. What would it cost to send ten $5 mail orders? one $50 mail order? How much more would the former cost than the latter? What per cent more?

4. A man who has a bank account prefers to pay a bill of $42.50 by check rather than by a postal money order. Why?

5. Try to obtain information about express money orders and their cost if there is an express office in your town. If you can obtain their rates, find the cost of sending the amounts in example 1 by express money orders.

6. See if you can obtain rates for paying money by telegraphic money orders, and if so, find the cost of sending $50; $75; $100; and $25 to various places.

7. Try to obtain information at your post office concerning the cost of foreign postal money orders, then make up examples and find their costs to different countries.

8. Go into your local bank and ask them to explain a bank draft to you. A man wishes to send \$1000 by a bank draft. If the bank charged him $\frac{1}{10}\%$ for the draft, as a fee, how much would he have to deposit with the bank to cover the total cost?

Review

1. Write a check on the Second National Bank of Mundale, N. Y., for \$150, payable to William Brown and signed by Henry Doe.

2. Try a 4th trial of some standard test in a preceding chapter.

3. Make another trial of the standard test in this book which has caused you the most trouble.

4. An automobile selling for \$1300 is reduced 15%. What is the price after the reduction?

5. A clothier bought $1\frac{1}{2}$ dozen caps @ \$1.00 each and sold them for \$1.50 each. What per cent of margin did he make on the cost? on the selling price?

6. The selling price of a certain article of furniture to the retailer was \$70. If the manufacturer's catalogue price gave a discount of 30%, what was the catalogue price?

7. Of what whole number is 36, 3%? 48, 4%? 50, 50%? 72, 25%? 100, 200%? 200, $\frac{1}{2}\%$?

8. A man is now paying \$66 a month rent for a house. This is 10% more than he was paying a year ago. How much was his monthly rent one year ago?

9. The marked price of a suit in a store window last week was \$45. The new price of the same suit is \$30. What rate of discount is being given?

10. Some of the longest tunnels in the world are the Hoosac, $4\frac{3}{4}$ miles; Loetschberg, Switzerland, $9\frac{1}{4}$ miles; St. Gotthard in the Alps, $9\frac{1}{3}$ miles; Simplon in the Alps, 11 miles; Gunnison, Colorado, 6 miles; Otira, New Zealand, $5\frac{1}{3}$ miles. Make bar graphs to represent these lengths.

The First Electric Engine to Enter the Simplon Tunnel

Mental Review

Write the answers to the following:

1. 7% of \$80 =

2. .04 × \$500 =

3. 60% of 40 =

4. .025 × 1000 =

5. 16 − ($\frac{1}{2}$ of 14) =

6. 15 ÷ ($\frac{1}{2}$ of 10) =

7. 75% of 24 quarts =

8. 50 − (10% of 50) =

9. 60 + ($16\frac{2}{3}$% of 240) =

10. 7 is ——% of 42.

11. Of what number is 30, 10%?

12. 20 + (15% of 20) − (5% of 20) = ?

13. 50 is $\frac{5}{6}$ of what number?

14. What is 20% more than 35?

15. $\frac{1}{2}$% more than 200 is what?

16. 400% of 4 more than 6 is what?

17. 50 is what per cent of 80?

18. 4 is what per cent of 2?

19. 10 is what per cent less than 20?

20. 50 is what per cent more than 40?

21. If 3 bananas cost 10¢, how many can you get for 40¢?

22. If 4 eggs cost 28¢, how much for 12 eggs?

23. $\frac{1}{3}$ of 15 = $\frac{1}{2}$ of what?

24. $\frac{1}{5}$ of 60 = ? of 24.

25. $\frac{3}{4}$ of 60 = ? of 180.

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER X

In this chapter you have learned that the foundation of the banking business is interest. You have had practice in some common banking customs. At the same time you have discovered how important to every one of us who deals with banks is skill in the fundamental operations of mathematics. You cannot afford to make mistakes with your own money or with any one else's funds.

Now test your skill with the End-of-Chapter Trial of the seven tests of Chapter X. Remember: out-of-school-time; improved record for each trial; 100% honor record in your work.

CHAPTER XI

EVERYDAY BUSINESS PAPERS AND ACCOUNTS

Now that we have laid the foundations of business with decimals, percentage, and interest, we are ready for more practice with other tools of business. We shall run across a new term or two, but all the work will be simply a further application of the New Mathematics which we have already learned.

Bills. Invoices. Statements. — Whenever we buy without paying cash, we usually get a *bill* for our purchase. Merchants, contractors, and others who do business usually send out bills to their customers at stated times. Harold T. Sullivan of Milwaukee, Wis., built a garage for Benjamin Somers. After it was completed he sent out the following bill:

HAROLD T. SULLIVAN
Contractor & Builder
186 LASALLE ST., MILWAUKEE, WIS

Mr. Benjamin Somers
Auburndale, Wis

Terms: 30 days net. Dr.

Aug. 12, 1927	To building garage as per agreement	$425	00

Bills are of various forms, but they always give the names of buyer and seller, the amount due, and the date. Sometimes the terms and the discount for cash are also given. " 30 days net " means that the bill may be paid any time within 30 days. " 2% 10 days " means that 2% may be deducted if the bill is paid within ten days.

Most wholesalers give a *discount* (see pages 215–221) as shown in the following bill.

JERSEY CITY, N. J., *Apr. 15, 1927*

Mr. S. Barabas
53 Beacon Ave., City

Bought of **H. WILLIAMS & CO.**

TERMS: Net 30 days, 2% 10 days.

1 doz.	V. S. P. Sweaters @ 10.00	120	00		
	25%	30			
				90	00

Invoices are lists. When the wholesaler sends to the retailer a list of items that the latter buys, together with the price and the discount that are offered, this list is called an *invoice* of the goods. An invoice is a list, not necessarily a request to pay. On the other hand a bill is a request to pay, and it may or may not contain a list of items.[1]

---

[1] Look up *bill* in the dictionary. Because it has so many different meanings business men commonly use the word *invoice* for it.

Statements are bills, sent out monthly, giving the dates and amounts of the month's bills and their total.

COMMITTEE WORK ON BILLS AND CHECKS

The Steering Committee and teacher should divide the class into five committees, appoint a chairman for each committee, and assign one of the following five wholesalers' invoices to each committee. The work of each committee should be done in a section of the classroom reserved for it, in the Library, or in some other room available at the time. Each chairman should preside over the committee discussion of how to make the bill and the check. Then each member should make out the bill and the check on papers cut to proper size.

The Chairman should select the best bill and check of his committee and turn these into the Steering Committee, who in turn should select the best bill and check of all committees. The winning papers should be placed on the class bulletin board.

Exercise

(*a*) Make out bills for each of the following.

(*b*) Write a check to pay each bill. (Use any names you wish.)

1. 2 dozen chairs at $50 a dozen; 3 dozen chairs at $30 a dozen; 3 single cots at $15; less 20%.

2. 3 bbl. of sugar, 640 lb. @ 4¼¢ a pound; 2 bbl. flour, 392 lb. @ 5¢; 453 lb. tea @ 45¢; 82 packages shredded wheat @ 12½¢; less 2%.

3. 4 pianos @ $460; 2 pianos @ $375; 5 pianos at $320; less 33⅓%, 25%, and 5% for cash.

4. 1 dozen baseballs @ $24 a dozen; 11 sweaters @ $15; 6 football head guards @ $6; less 10%.

Receipts and Vouchers. — When a bill is paid, the one who receives the money should give the payer a *receipt*. This is usually written on the bill. Below is the receipt for Mr. Sullivan's bill on page 278. Mr. Somers paid the bill Aug. 20. Mr. Sullivan then returned the bill to Mr. Somers with a receipt signed at the bottom as you see. The bill is now called a *receipted bill* or simply a *receipt.*

HAROLD T. SULLIVAN
CONTRACTOR AND BUILDER
186 LASALLE ST., MILWAUKEE, WIS.

Mr. Benjamin Somers
Auburndale, Wis.

TERMS: 30 days net. DR.

Aug. 12, 1927	To building garage as per agreement *Received payment, Aug. 20, 1927* *Harold T. Sullivan*	$425	00

On this bill the expression "30 days net" means that no interest will be charged if the bill is paid within 30 days. The abbreviation "Dr." stands for Debtor, and means that the amount stated under it is *owed.*

If Mr. Somers pays this bill by check, he has three evidences that he has paid it:

1. The stub entry in his check book.
2. The receipted bill.
3. The canceled check, which the bank returns to him.

A canceled check or a receipt, in fact any paper which proves a payment, is called a *voucher*. But in usual practice canceled checks are called vouchers, and receipts are called merely receipts.

When a business man receives a check in payment of a bill, he usually sends no receipt, as the canceled check when returned by the bank shows that the money has been paid. Thus a canceled check is a voucher, a kind of receipt.

There is another form of receipt, used when no bill is made out (rendered). Here is a receipt for insurance, where no bill was rendered.

EXETER, N. H., *April 4, 1927*

Received from ——— *Alex Murphy* ————————
Thirty-six dollars for payment of insurance in full

$36.00 *Millard S. Reynolds.*

The receipt states the amount paid both in words and in figures, and usually also the purpose for which the money is paid. In the case of payments made in currency, that is, not by check, it is advisable to have a receipt like the one shown above.

If you pay a part of a bill, then the receipt states that a certain amount has been received *on account.*

DRAMATIZING BUSINESS TRANSACTIONS USING WHOLESALE INVOICE, RETAIL BILL, CHECK, STUB ENTRY, RECEIPT, AND VOUCHER

Directors of Dramatization: Steering Committee and Teacher, who prepare the business transaction.

First Part: prepare business forms (work done prior to class period).
Wholesalers: Two pupils to prepare invoice to retailer
Retailers: Two pupils to prepare bills to the purchaser Check Clerks: Two pupils to prepare stubs and checks from retailer to wholesaler and from purchaser to retailer
Receipt Clerks: Two pupils to prepare receipts from wholesaler to retailer and from retailer to purchaser

Second Part: Dramatization in charge of Directors
Players: Wholesaler's Bookkeeper (invoice to retailer)
Retailer's Bookkeeper (bill to purchaser)
Purchaser (stub and check to retailer)
Wholesaler's Cashier (receipt to retailer and indorsement of retailer's check for bank deposit)
Retailer's Cashier (check to wholesaler, and stub, receipt to purchaser, and indorsement of purchaser's check)
Bank Clerk (mark both checks "paid" and return as vouchers)

At the close of all transactions the class should decide to whom each paper belongs.

Writing Receipts

1. Suppose that Horace Fowler has purchased a bicycle from you for ten dollars. Write a receipt such as you should give him when he pays for the bicycle.

2. Suppose that you had paid a store account of $5.65 due Joseph Waters, the merchant. Write the kind of a receipt that you would like to have Mr. Waters give you.

3. Imagine that you owed Henry Eveland $10.50 and paid him $7.50 on account. Write the form of a receipt that you would like to have him give you when you paid him.

4. Gerald Martin owed Harry Burgin $150. He had owed this bill for 1 year from June 2, 1926. Mr. Burgin's store charged interest at the rate of 6% per year on all bills that were not paid within 30 days. Find the amount of money that Mr. Martin would owe Mr. Burgin on June 2, 1927. See if you can write the kind of a receipt that you would like to have if you were Mr. Martin.

Promissory Notes. — Sometimes a man has not the money to pay for goods he wants to buy. If the seller does not wish to lose the sale, he offers the goods to the purchaser on condition that the latter will promise to pay him later. For instance, in the winter or spring farmers often need tools, feed, or seed for which they cannot pay till the crops are harvested in the fall. Merchants sell goods in the spring on the farmer's promise to pay when the crops are marketed. This promise when put in writing is called a *promissory note*, or just a *note*.

A note is both an acknowledgment of the debt and a promise to pay. It should state the date, the time it is to run, and the rate of interest. The following is the form of a promissory note.

$ ...*300.00* . Factoryville, Pa., ... *May 15, 1927*....

......*Three months*.. ... after date ..*I*.... promise to pay

.... *Henry Sellers, or order*. . . ..

Three Hundred . . ..Dollars

For value received, with interest at six per cent.

............*Henry Sullivan*............

The person to whom the money is to be paid is called the *payee*. Who is the payee in the above note?

The one who is to pay the money is called the *maker*. Who is the maker? What rate of interest must the maker pay?

Upon what date is the note due?

How much will be due Mr. Sellers, including interest, when the note falls due?

Indorsing Notes. — In the transaction above when the note falls due, Mr. Sellers may desire to have Mr. Sullivan pay it to some person other than himself. Suppose that he owes Thomas Crane $500, and wishes this note paid to Mr. Crane as part payment. In that case he writes an indorsement on the back like this.

Pay to
Thomas Crane
Henry Sellers

This indorsement makes Mr. Sellers liable for payment of the note. That is, when the note falls due, if Mr. Sullivan fails to pay it, Mr. Sellers must. For this reason we should be cautious about indorsing notes.

Mr. Sellers must not indorse the note to Mr. Crane, unless it bears the words " or order," or " to the order of." These words make it *negotiable*, that is, payable to the person to whom the payee indorses it. If these words are omitted, the note is not negotiable.

If nothing is said about interest, in most states the note will bear interest at the legal rate. If there is to be no interest, the words " without interest " should be written in the note.

A note is often called an I. O. U. Can you explain why? Who should have possession of a note until it is paid?

Promissory Note Practice

1. Make out a note for $100 with John Evans as payee and yourself as the maker. Date it May 3, 1927. Make it payable four months from date, bearing interest at 5%, and negotiable.

2. Indorse this note, making it payable to Harry Stephens.

3. Find the interest to be paid on this note when it is due.

4. How much would you have to pay Harry Stephens at the end of the four months?

5. Make out a note for $50 which Henry James owes you as payee. Date it March 1, 1927 and make it due in 2 months with interest at 6%.

6. How much interest would Mr. James owe you at the end of that time?

7. How much would he have to give you in order to pay you in full at that time?

8. Make out a note for $1500 which John Brown owes. Have it due in 6 months at 4%. Date it April 1, 1927. Make Harry Howard the payee.

9. When is this note due?

10. How much will it take to settle this note in full when it is due?

FOURTH TRIAL OF I. R. B. TESTS OF CHAPTERS VIII AND IX

Before you proceed very far with Everyday Business Papers and Accounts it will be well, if you have not already done so, to catch up your I. R. B. records of the Fourth Trials. Turn back to Chapters VIII and IX and take in out-of-school-time the I. R. B. tests of these two chapters. Enter your new scores in the Fourth Trial column. Do not be disheartened if you do not excel all former scores; try again till you do; read directions of the Fourth Trial exercise in Chapter V.

Accounts. — We have now become familiar enough with bills, receipts, checks, vouchers, and notes, so that we ought to be able to put them to work in everyday practice. This can be done best by keeping *accounts.*

While in school you will be busy with your personal accounts and in raising and spending class and school funds, in school savings accounts, and in many other relations with the business world. People will respect you more if you convince them that a Junior High School pupil knows business customs well enough so that he can use the conveniences of the commercial world.

In the very first chapter Jack Wood kept a cash account. A few pages back we opened an account at a bank. We are now going to study some other accounts.

Balancing Accounts. — Accounts are best kept in a book called a *cash account* which is ruled like the illustration below. This is Mabel Westcott's account.

CASH ACCOUNT

1925		*Mabel Westcott*	Receipts		Expenditures	
Aug.	2	Cash on hand	2	75		
Aug.	3	Weekly allowance from Father	2			
		Movies				25
		Street car fare				14
Aug.	3	Treated Clara to ice cream				30
Aug.	4	Picked berries for Aunt Anna	1			
Aug.	5	Hair bobbed				50
Aug.	6	Stockings				75
Aug.	8	Sewing on buttons for Uncle Henry		25		
		Dancing lesson			1	00
		Totals	6	00	2	94
		Balance			3	06
Aug.	9	Cash on hand	3	06		

At the end of every week Mabel adds the receipt column. Then she adds the expense column, and by subtracting finds out how much money she ought to have on hand. Then she counts her money, and if the results agree, her accounts *balance*. Then she enters this amount under the expense column and draws the line as shown. What is the total of each of the columns?

If the accounts balance, these columns must agree. A real bookkeeper would write the word *balance* in red.

Now she is ready to open the account for the next week by entering $3.06, the amount of last week's balance, as cash on hand.

Making Accounts Balance

Make out a weekly account and balance it for each of the following:

1. Eleanor Smith.

Sept. 6, 1927, cash on hand $1.86. Received: Sept. 8, $.25 for washing dishes for mother; Sept. 10, caring for cousin Ralph $.60; Sept. 12, birthday gift from father $2.00. Spent: Sept. 7, picnic $1.60; Sept. 9, ice cream soda $.15; Sept. 10, Scout dues $.25; Sept. 11, hair ribbon $.20. Balance this account Sept. 12.

2. Harry Emerson.

Oct. 4, 1927, cash on hand $2.15. Received: Oct. 5, $.35 for delivering meat; Oct. 6, $.75 for running errands; Oct. 7, $.50 from father as a weekly allowance; Oct. 8, $.96 for selling Post; Oct. 10, $1.25 for delivering packages. Spent: Oct. 4, $.05 for church; Oct. 6, $.15 for movies; Oct. 9, school athletic dues $.25. Find the balance Oct. 11.

3. Norman Tymeson.

Nov. 9, cash on hand $5.67. Received: Nov. 9, from father $2.00; Nov. 10, work for father $.75; Nov. 12, errands for mother, $.25; Nov. 14, helping father wash car $.30. Spent: Nov. 10, candy $.10; Nov. 11, movies $.15; Nov. 14, a football $3.75. Balance account Nov. 16.

4. Mary Ralston.

Dec. 7, 1927, balance on hand $1.35. Received Dec. 7, $.25 for helping in house; Dec. 8, tending a baby $.25; Dec. 9, running errands $.15; Dec. 11, helping a married sister $.35; Dec. 12, helping in a store $1.25. Spent: Dec. 8, carfare $.14; Dec. 10, soda $.10; Dec. 11, movies $.15. Find balance Dec. 14.

5. Glance over the following account and estimate Henry Martin's balance.

Dec. 1, 1927 cash on hand $2.75. Received: Dec. 1, $1.00; Dec. 3, $.75; Dec. 5, $1.50; Dec. 7, $.25. Spent: Dec. 2, $.50; Dec. 4, $.15; Dec. 5, $.10; Dec. 6, $.05. Dec. 6, $.05. Balance Dec. 8, $5.40. Make this cash account and check Henry Martin's balance.

I. R. B. RECORD

In your I. R. B. enter one of each of the following business papers, giving your own original illustrations: invoice, bill, receipted bill, receipt, check, promissory note, and cash account.

Household Accounts — The Budget. — When John Bowman and Mabel Gray were married and wished to go to housekeeping, they had to solve many problems which may come to each of you some day.

John's salary was $300 a month, and he had trouble to make it last. The first year he and his wife had to give up some things they really needed because of unwise buying. The second year they made a plan like this for each month's salary.

ALLOWANCE

For rent	$ 70
For heat and light	20
For telephone	4
For food	50
For clothing	35
For books and magazines	2
For amusements and recreations	10
For church and charity	10
Fund reserved for illness	15

Upkeep of automobile	14
Miscellaneous	10
Savings account	60
Total	$300

Such a plan for one's income is called a *budget*. After John and his wife made their budget they had no more trouble, for they kept all expenses within the limits that they had set.

What per cent of John's income went for rent? for food? for clothing?

How much will it cost to maintain his automobile for one year?

If their magazines cost $12 a year, how many new books can they buy each year at $1.20 each?

If neither is sick during the entire year, John will add that fund to his savings account. How much would he add?

If he puts in the bank only the amount stated in the budget, how much will he have in the bank after 10 years? (Take no account of interest.)

Mention some of the expenses that you think should be classed as miscellaneous.

What is the ratio of his clothing bill to his rent? of his automobile upkeep to his food bill? Draw bar graphs to represent all of the above expenses.

Buying or Renting. — John finds he can buy the house where he lives for $9000.00. If he owns the house, he will have to pay taxes amounting to $150 a year; water rent of $12 a year; and the interest on the money he borrows to pay for the house, which at 6% would be $540 a year. He estimates that $100 a year will be needed for upkeep and repairs and $15 a year for insurance.

How much will it cost him a year to own the house?

How much did it cost him a year to rent it?

What per cent of the cost value of the house is its annual rental?

The annual taxes are what per cent of the value of the house?

The insurance rate on this house was $\frac{1}{3}$% of its value a year. What was the cost of insuring it? What was the ratio of insurance cost to the purchase price of the house?

The Business of Insurance. — We have just spoken of insurance. Fire insurance may best be explained by an example. John's friend Edward Roberts had a house that he valued at $10,000. He had it insured against fire for $8000. This means that the insurance company would pay him $8000 if the house were completely destroyed by fire. For this protection he had to pay the insurance company each year $\frac{1}{4}$% of $8000. Then the company gave him a printed contract called a *policy*, which stated the conditions of the agreement between them.

The $8000, the amount for which the house was insured, is called the *face of the policy*.

The amount paid each year, $\frac{1}{4}$% of $8000 or $20, is called the *premium*.

It is usually not possible to secure insurance on a building for an amount greater than, or even equal to, its real value. Can you suggest why? If Mr. Roberts' house were only partially destroyed, he could not get the whole $8000. The insurance company would then send a man called an "insurance adjuster" to look over the damaged building and arrange with the owner the amount of payment which he felt the insurance company ought to pay.

A SCHOOL DESTROYED BY FIRE

Frequently the premium is calculated as so many cents on a hundred dollars. For instance, in the above example suppose that the premium had been at the rate of 25 cents per hundred dollars, then we should have the following

Solution: \$8000 ÷ \$100 = 80 hundreds of dollars.
80 × \$.25 = \$20, premium.

So we see that insurance also is only another application of percentage.

I. R. B. Test [**1**]. Your Steering Committee and teacher should determine a standard time for this test and for the other two I. R. B. tests of this chapter when you come to them. Prepare Chart for Chapter XI (3 tests) and enter First Trial scores. Copy and complete the following:

AMOUNT OF INSURANCE	RATE OF PREMIUM	PREMIUM
\$6,000	$\frac{1}{2}\%$	
\$10,000	50¢ per \$100	
\$10,000	$\frac{2}{5}\%$	
\$12,000	40¢ per \$100	
\$6,500	45¢ per \$100	
\$5,000	43¢ per \$100	

Buying Fire Insurance

1. How much will it cost me to insure my barn for \$2400 at an annual premium of $\frac{5}{8}\%$? (Note that $\frac{5}{8}\%$ means $\frac{5}{8}$ of 1%.)

What per cent is the same as $\frac{3}{4}$? Is that the same as $\frac{3}{4}\%$? What is the difference between the two on \$100.

2. Find the annual premium on a policy of \$5600 if the rate is \$.65 per \$100.

3. If the rate is \$1.30 a \$100, what will it cost to insure the contents of a store valued at \$18,000?

4. Try to explain why the rate of insurance might be much more in one town or city than in some other town or city. Explain why it might be much more in some parts of a city than it might be in other parts of the same city.

5. What will it cost to insure a private frame garage if the value is allowed at \$300 and the insurance costs \$4.00 per \$1000?

6. If the rate on a framed shingle-roof boarding house is 52 cents per \$100, what will it cost to insure it for \$9600?

7. The rate on a brick metal-roof private house located within 800 feet of a fire hydrant in a city with excellent fire protection is 15 cents per \$100. If it is insured for \$12,500, what will be the annual premium?

8. The annual rate on a two-family house of the same construction in the same city, and under similar conditions as example 7, was 17¢ per $100. What would it cost to insure it for $16,800?

9. Mr. Harris insured his factory, which cost $40,000, for $\frac{5}{8}$ of its value at an annual rate of $\frac{7}{8}\%$. For how much was it insured? How much was the annual premium?

10. The annual rate on a piece of property was 80¢ per $100. If the annual premium was $65.60, for how many hundreds of dollars was it insured?

Making the Garden Pay. — John and Mabel wanted a garden because they both liked fresh vegetables. Whether a back yard garden will be profitable or not depends on the size of the back yard, the nature of the soil, and the amount of leisure time one has to work it.

John found room for a garden 50 feet long and 25 feet wide. He paid a man $3 to spade it, but most of the work he did himself. He spent $3 for fertilizer and $1.40 for seeds and plants. He paid a boy 50 cents to pick insects off his vines.

He raised $\frac{3}{4}$ bushel of peas, $\frac{1}{2}$ bushel string beans, 2 dozen cucumbers, 50 heads of lettuce, 50 bunches radishes, 20 pounds of tomatoes. The average prices of these vegetables in the city market was as follows:

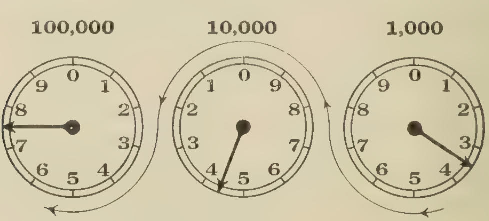
Peas, 30¢ a peck.
String beans, 30¢ a peck.
Cucumbers, 5¢ each.
Lettuce, 5¢ a head.
Radishes, 2 bunches for 5¢.
Tomatoes, 10¢ a pound.

What did his garden cost him?
What was the value of his crop?
How much did he save by raising these vegetables?
Do you think that it paid him to have a garden?

Note. Let the class discuss this last point freely if time permits.

Buying Gas and Electricity. — John's house had a gas range and electric lights. Both gas and electricity were measured by meters in the cellar. To make sure that the bills were always correct John learned to read the meters.

Buying Gas. The gas meter had a dial like this:



CUBIC FEET

John noticed that the right-hand dial had numbers running in the order of the numbers on a clock, and the hand ran in the same direction as the hands of the clock. The numbers of the middle dial ran in the opposite direction and its hand moved in the opposite direction to the hands of a clock. The left-hand dial was like the right-hand dial.

The right-hand dial measures in hundreds of cubic feet up to and including 1000 cubic feet, so the illustration shows 3 hundred cubic feet. The middle dial denotes thousands of cubic feet up to and including 10,000 cubic feet, so the diagram shows 4 thousand cubic feet. The left-hand dial shows 10 thousands of cubic feet up to and including 100,000 cubic feet, so this dial shows 7 ten-thousands or 70,000 cubic feet. When the hand points between two figures, take the lower one. Thus we have for a total reading:

$$70000 + 4000 + 300 = 74{,}300$$

This means that 74,300 cubic feet have been measured on this particular meter, just as 23,400 on the speedometer of an automobile would mean that 23,400 miles had been traveled by that automobile.

Point to where the hands should be to measure 45,600 cubic feet of gas.

John watched the gas man read the meter and determine how much gas has been used. He did it as follows:

Reading of meter April 10, 1927	56,700
Reading of meter March 10, 1927	53,300
Gas used during the month	3,400 cubic feet

The gas man figured it so easily that John thought he would practice a bit, so that it would soon be a simple matter for him. So he set himself a number of problems to solve.

Illustrative Example 1. Find the cost of the gas used in a certain home from the above readings at \$1.15 a thousand cubic feet.

Solution: $3400 \div 1000 = 3.4$
$3.4 \times \$1.15 = \$3.91$

Illustrative Example 2. Find the amount of the above gas bill if a man paid it within 10 days and received $\frac{1}{10}$ off for prompt payment.

Solution: 10)\$3.91
.39 deduction for cash in 10 days
\$3.52

Is it wise to pay promptly if a deduction is allowed for it?

Checking Gas Bills

Explain where each hand should be to read each of the following:

1. 27800 cubic feet.

2. 32400 cubic feet.

3. 96300 cubic feet.

4. 9600 cubic feet.

5. 12000 cubic feet.

6. 600 cubic feet.

7. 10000 cubic feet.

8. Can you tell where the hands would be February 1 if 78,000 cubic feet had been measured up to January 1 and 2300 cubic feet had been consumed during the month of January?

Find the amount of gas consumed during the month from the following readings if the latter number denotes the reading at the end of the month:

9. 57,600 – 60,300.

10. 24,800 – 28,300.

11. 89,500 – 92,400.

12. 67,200 – 71,100.

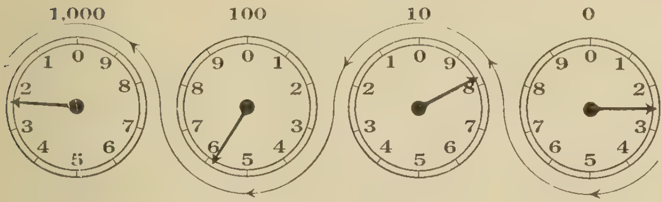
13. Find the cost of the gas consumed in each of the examples 9 to 12 at $1.10 a thousand cubic feet.

14. Find the cost of each of the above in case 5% deduction were allowed if a person paid cash within 10 days.

15. Bring a gas bill to school if you can do so and let the class verify it.

16. With your compasses draw three diagrams of a gas meter and indicate on them the readings given in examples 1, 2, and 3.

Buying Electricity. John found the electric meter harder to read. Electricity is measured in *kilowatt-hours* on a dial like the one which appears at the top of the next page.



KILOWATT HOURS

John noticed that the hand of the right hand dial runs in the same direction as the hands of a clock. The next hand runs the opposite way, as does also the left-hand one. You must remember this in reading the meter.

One complete turn on the right-hand dial denotes 1 kilowatt-hour, and the figure 1 on that dial denotes .1 of a kilowatt-hour.

One complete turn on the next dial to the left denotes 10 kilowatt-hours.

One complete turn on the next dial to the left denotes 100 kilowatt-hours.

One complete turn on the left-hand dial denotes 1000 kilowatt-hours. What, then, does the meter read in the above drawing?

Checking Electric Bills

1. Tell the position of the hands to denote 258.2 kilowatt-hours.

What will be the position of the hands to denote

2. 356.7 kilowatt-hours?

3. 472.4 kilowatt-hours?

4. 569.3 kilowatt-hours?

5. 126.8 kilowatt-hours?

6. 753.0 kilowatt-hours?

Find the number of kilowatt-hours that had been used during the month from the following. The second number denotes the reading at the end of the month.

7. 453.6 —— 464.0.

8. 325.7 —— 340.9.

9. 567.4 —— 600.6.

10. 892.4 —— 919.7.

The following represent kilowatt-hours. Find the cost at 9 cents per kilowatt-hour.

11. 24.5.

12. 16.7.

13. 12.3.

14. 17.2.

15. 29.1.

16. 32.6.

Very large users of electricity are often given a lower rate than small users of electricity. Find the monthly cost of each of the following at 5 cents a kilowatt-hour.

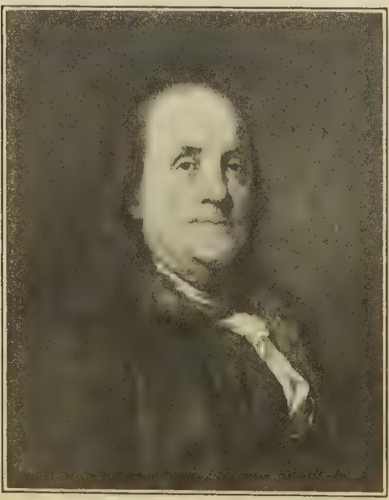
17. 325.6 kilowatt-hours.

18. 672.4 kilowatt-hours.

19. 562.7 kilowatt-hours.

20. 956.4 kilowatt-hours.

21. Bring some electricity bills to class if you can and verify the cost of each.



Benjamin Franklin

REVIEW TRIAL OF I. R. B. TESTS, CHAPTER X

In your I. R. B. you have entered two sets of scores for the tests of the preceding chapter. Now take the same seven tests a third time, stick-to-it to improve former records and enter the third scores in the Review Trial column. (Out-of-school exercise.)

The Business of Economical Buying. — A great American, Benjamin Franklin, wrote many wise sayings. Here is one of them: "He who buys what he has no need of will ere long sell his necessaries." Suppose you see some lemons at 10 cents a dozen. You don't need any lemons, but you like a bargain. You buy them, forget them, and they spoil. You have lost 10 cents.

It is cheaper to use the "cash and carry" system than to order by telephone and run an account. Can you see why the cost of delivery wagons and extra bookkeepers must be covered by the price of the goods?

Another way to save money is to buy only what is in season. Asparagus, peas, strawberries, all are cheap in season, and they taste better at that time. Some fruits, like oranges and bananas, cost about the same all the year round. These are the ones to buy when others are out of season.

Saving Money

1. Mrs. Bowman needed a new coat. She saw one that she liked in a store window for $10 down and $10 a month for 6 months. Later she saw one just like it in a cash store for $55, so she bought it. How much did she save by paying cash for it? What per cent did she save?

2. Mabel's grocery bill used to average $20 a month, but by carrying her groceries and paying cash she was able to save 15%. What is her average bill now?

3. John bought 6 pair of hose at a sale for $5.40, thus saving 10% of the usual price. What was the usual price per pair?

4. Goods that will keep should be bought in large quantities. Why? Soap can be bought at 6 cents a cake or 66 cents a dozen. Canned goods can be bought at 18 cents a can or \$1.80 a dozen cans. Sugar can be bought at 6 cents a pound or at \$5.25 a hundred pounds. A 12½-pound sack of flour can be bought for 50 cents or a 49-pound sack for \$1.75.

What per cent is saved by buying:

Soap by the dozen cakes?

Canned goods by the dozen cans?

Sugar by the hundred weight?

Flour in 49-pound sacks?

Would it be better for John and Mabel to buy bananas by the bunch and eggs by the crate? Why? Should a restaurant owner buy them that way?

TEAM WORK AND COÖPERATION IN A REVIEW TEST

Let the Steering Committee and teacher divide the class into four teams, appointing a captain and lieutenant for each team, and apportioning the pupils as evenly as possible according to previous records.

Spend one class period in working as many of the following 25 examples as possible; the captains and lieutenants acting as coaches. Then let each pupil complete the exercise in out-of-school-hours. On the following day return papers to team captains. While the captains and lieutenants are checking and scoring the papers of their own teams, let the Steering Committee assist all team members with examples which caused difficulty. Finally the Steering Committee will determine team averages and the winning team.

If you and your Steering Committee wish, you may modify the above procedure in part or in whole. One of the duties of the Steering Committee is to adapt these suggestions of contests and games to your class work.

Exercise

1. What per cent would be gained on the cost if goods are sold for $30 which cost $24? What per cent would be made on the selling price?

2. A single-way ticket between two cities costs $35. A round-trip ticket between those cities costs $60. What per cent would be saved by buying a round-trip ticket instead of two one-way tickets?

3. An insurance company charged $60 for insuring a man's building for $15,000. What rate of insurance was charged?

4. If a liter of water weighs 1 Kg. = 2.2 pounds, what will a 40-quart milk can full of water weigh in pounds? (Do not consider weight of can.)

5. A dealer bought an automobile for $1500 with discounts of $\frac{1}{4}$ and 10%. What did it cost him?

6. I sell goods for $2.25 at a loss of 75 cents. What per cent do I lose on the cost?

7. How much would it cost me to insure a cargo of grain for $40,000 at $1.40 a $100?

8. The product of two numbers is 1.125. One of the numbers is .25. What is the other number?

9. The difference between two numbers is 4.162. The smaller number is .345. What is the other number?

10. Anna missed 3 examples out of 50. What per cent did she get right?

11. The caloric fuel value per pound of eggs is 765 while that of milk is 1520. How much less is the former than the latter? What per cent less? What is their ratio? Express these relations by means of bar graphs.

12. Potatoes contain 2.2% protein food and .1% fat. How much of each of these in 1 bushel of 60 pounds?

13. A certain Ford automobile dealer sold a certain model of the car for $430.99 cash, or $120 cash and 12 monthly payments of $32 each. How much would I pay for the car under the latter terms? What per cent of the cash price is this amount?

14. I. R. B. Test [2]. Record First Trial scores. In the following tabulation carry your division out to at least 3 decimals. For instance, if an answer should come out .036, we would write 3.6 (per cent) in the last column. Complete:

Year	Expended in Advertising	Volume of Sales	Per Cent Advertising Is of Sales
1914	$2,500	$256,860	
1920	55,900	4,301,312	
1921	218,338	8,370,637	
1922	434,871	11,452,373	

15. A man obtained 85 pounds of milk from his cows. If this milk contained 3½% butter fat, what was the total butter fat in that amount of milk?

16. If 18 out of 24 grains of corn sprouted in a certain seed test, what per cent sprouted?

17. If one half gallon of ice cream will serve sixteen, how many quarts are needed for a party of twenty-four?

18. At a special sale, dresses were marked as selling at one third off. For what was a dress formerly sold that is now selling at $36?

19. If 24 quarts of raw fruit will can 16 quarts, how many quarts will 36 quarts of raw fruit can?

20. If $500, the base, is the whole, and 20%, the rate per cent, is the fractional part of the whole, what is the percentage or part of the whole?

21. If 16⅔% represents the fractional part of the whole, and $32 represents the part of the whole, what is the whole?

AN ORANGE GROVE

22. If John bought 10 dozen oranges at 40 cents a dozen and sold them at 5 cents each, what per cent did he make on the cost?

23. I. R. B. Test [3]. Enter First Trial scores. Complete the following:

BASE OR WHOLE	RATE OR FRACTIONAL PART	PERCENTAGE OR PART OF THE WHOLE
$600	25%	
$256.25	20%	
	12½%	$40
	37½%	$45
$200		$20
$17.50		$3.50

24. Two thirds of the length of a pole is in the ground, $\frac{1}{5}$ of it is in the water, and 12 feet is in the air. What is the length of the pole?

25. Draw lines of the following lengths without the aid of your ruler, then use your ruler and see how well you have estimated these lengths:

$\frac{1}{4}$ ft., 1 ft., $\frac{3}{4}$ ft., 5 in., 6 cm., 10 cm., 16 cm., 5 dm.

END-OF-CHAPTER TRIAL OF I. R. B. TESTS OF CHAPTER XI

In this chapter you have extended your knowledge of the common commercial papers and accounts which so simplify the Everyday Business of all of us. Business men will respect you for your ability to use these conveniences without error. They very much admire girls and boys who can be trusted to work faithfully and with clean records.

So try again the three I. R. B. tests of Chapter XI; work faithfully until you surpass your First Trial scores: "come clean"; and be worthy of the confidence of American 100% Business Men.

CHAPTER XII

GENERAL BUSINESS SURVEY

Business in General. — When we began to use the New Mathematics in business, we saw (page 163) how universal business is. We all have some business. So most of our problems are business problems. When we come to solve our business problems, we shall be surprised to see how many of them use only the simplest processes of mathematics. Probably more than three fourths of the calculating done by business men is in the four fundamental processes, fractions, and decimals. Of course decimals include percentage, margins, discount, commissions, interest, and many other things, but if we know decimals, we can do them all.

It will always help us, if we are quick and accurate at these simple things. To become quick and accurate we should use the New Mathematics wherever we can, especially in the algebraic solution of our problems.

Mathematics without Numbers. — Before we start our special exercises, it will help us if we review some of the general statements in the New Mathematics. We have seen how the formula introduced us to algebra. When we have a good hold on these general principles as expressed in formulas, it is easy to apply them to our particular problems.

Word Problems

After you have answered the *how* of the following examples write an answer using letters as a formula; for instance, " percentage = base times rate " could be written **p** = **br.** The substitution of letters for words in this way is algebraic shorthand.

1. If you know the base and the rate, how do you find the percentage?

2. The area of a rectangle and altitude being given, how do you find the base?

3. If the subtrahend and remainder are known, how would you find the minuend?

4. If the difference of two numbers is known and you know the smaller number, how is the larger number found?

5. The product and multiplier being given, how is the multiplicand obtained?

6. The cost and rate per cent of margin of gain on the cost being known, how is the gain in money found?

7. If you knew the cost and the loss, how would you find the selling price?

8. If the selling price and gain are known, how is the cost found?

9. The amount of money for which a building is insured and the rate per hundred dollars charged for the insurance being given, how would you find the premium?

10. If the sales of eight salesgirls were known for a single day, how would you find the average sales per clerk?

11. Given the dividend and the quotient, and the remainder which is more than zero, how shall we find the divisor?

12. The selling price and the cost are known. Tell how you would find the margin or gain per cent on the cost. On the selling price.

13. The sum of three numbers being given and two of the numbers, how would you find the other number?

14. If you know the gain on a dozen things, how do you find the gain on one?

15. If you know the cost and the selling price of one article, how do you find the gain on one? on fifty? on one hundred twenty-five?

The Shorthand Method. — This use of letters in formulas may be extended to other problems. In algebra a letter may stand for any number.

Now let us try using letters in some other kinds of problems. For instance, you wish to find out how many pounds of sugar at 6 cents can be bought for 48 cents.

Let n = the number of pounds.

Solution:

6 times $n = 48$

or briefly $6\,n = 48$

Therefore $n = 48 \div 6 = 8$

Check $8 \times 6 = 48$

Hence, we can buy 8 pounds of sugar.

More Shorthand

Find the value of n in each of the following equations. See how many of them you can do-in-your-head. Check your answers.

1. $6\,n = 30.$
2. $5\,n = 75.$
3. $4\,n = 48.$
4. $12\,n = 60.$
5. $8\,n = 72.$
6. $8\,n = 15.2.$
7. $9\,n = 16.2.$
8. $12\,n = 13.2.$
9. $15\,n = 30.$
10. $16\,n = 64.$
11. $\frac{1}{2}\,n = 8.$
12. $\frac{1}{4}\,n = 12.$
13. $\frac{1}{6}\,n = 24.$
14. $\frac{3}{4}\,n = 9.$
15. $\frac{5}{8}\,n = 30.$
16. $.05\,n = 10.$
17. $.03\,n = 12.$
18. $.08\,n = 2.4.$
19. $.07\,n = .42.$
20. $1.5\,n = 45.$
21. $.09\,n = 6.3.$

Solving Problems by Algebra. — We shall use algebra more and more as we go on with our mathematics, because it makes our work shorter and easier. It enables us to solve very easily certain types of problems which would be hard by arithmetic.

Illustrative Problem 1. An automobile travels 154 miles in 7 hours. How many miles per hour does it average?

Solution: We let m stand for the unknown number of miles here because m is a good shorthand for miles.

We then have $7\,m = 154$

Or $m = 22$ miles per hour.

Illustrative Problem 2. John is twice as old as his sister and the sum of their ages is 24 years. How old is each?

Solution: Let $x =$ the number of years in the sister's age.

Then $2\,x =$ the number of years in the brother's age.

Now x plus $2\,x$ may be added just like 1 qt. + 2 qt., and we have

$$3\,x = 24$$
$$x = 24 \div 3 = 8 \text{ years, sister's age.}$$
$$2\,x = 2 \text{ times } 8 \text{ or } 16 \text{ years, John's age.}$$

Check. $8 + 16 = 24$.

Algebra Problems

Solve the following problems by the algebra method.

1. A bicyclist rode 84 miles at an average speed of 14 miles per hour. How many hours did it take him?

2. The perimeter of a square is 24 inches. Find the length of one side.

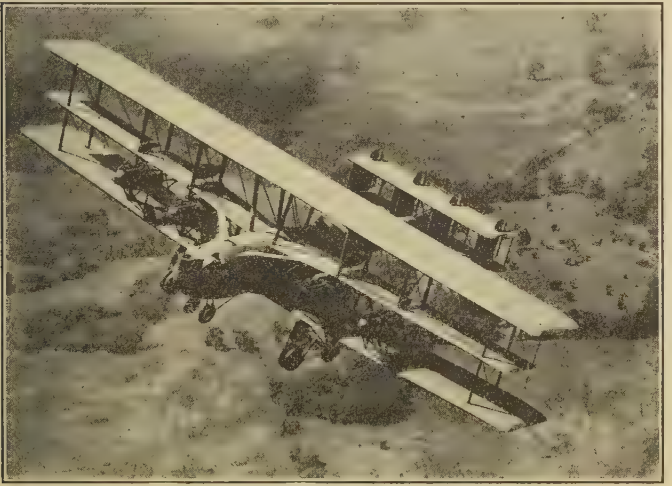
3. The area of a rectangle is 156 square inches. If the length is 13 inches, what is the width?

4. The area of a parallelogram is 160 square inches. If the height or altitude is 5 inches, what is the base?

5. One number is 4 times another number and their sum is 35. Find the numbers.

6. A man and his son earned \$6.00 one day. If the man earned 3 times the amount that his son earned, how much did each earn?

7. An airplane made 966 miles in 7 hours. Find the speed per hour.



A DREADNAUGHT OF THE AIR

This giant plane weighs 40,000 pounds. How many tons?

8. The area of a rectangle is 32.85 sq. in. If the length is 7.3 in., what is the width?

9. A man sold a house for \$9000 and made $\frac{1}{5}$ on the cost. What was the cost? *Hint:* Let x = the cost. Then $\frac{1}{5}x$ = the gain. $1\frac{1}{5}x = \$9000$.

10. A man sold an automobile for \$900 and lost $\frac{1}{4}$ on the cost. Find the cost.

11. A man received \$64 interest from some money that he had loaned. If he received \$18 interest on the sum per year, how many years had his money been out at interest?

12. A musician received $60 a week for 6 days of 6 hours each. How much does he receive per hour?

13. A line 40 inches long is divided into two parts, so that one part is four times the other part. How long is each part?

14. The length of a rectangle is three times its width and its perimeter is 48 feet. Find its length and its width.

Application in Review. — In problems like the preceding, various simple processes are used again and again. To help in our solutions we should practice these simple operations till we can do them with the utmost ease. For this reason we are going to take a series of timed tests in these operations.

ACCURACY AND SPEED REVIEW FOR I. R. B. TESTS [1] TO [8] INCLUSIVE

Each of the following Parts is a correspondingly numbered I. R. B. Test. Prepare your Chart for Chapter XII (8 tests). Take the first 6 tests at one time; either in or out-of-school as your teacher directs. First the Steering Committee should take the tests to determine standard times. Exchange papers and record on one another's papers the scores based on 100%. Your Steering Committee as you are taking the tests will announce the time scores which each of you makes. Raise your hand when you finish a test.

If you take the tests out-of-school, you may keep your own time scores and score your own accuracy. At the conclusion of all six tests in arithmetic the Steering Committee should average both the accuracy and time scores of the class. Graphs of these average scores should be drawn on the blackboard. Copy these graphs into your I. R. B. and draw corresponding graphs of your own scores. These graphs will make the I. R. B. tests [7] and [8]. In these two tests omit the time score but have the Steering Committee score your graphs for accuracy and neatness and enter these scores in the First Trial column.

I. R. B. Test [1].— Review of Fundamental Operations. Without copying the examples start at the bottom and see how long it takes you to climb the ladder to the top. Write answers for each space on another paper.

$9\overline{)72}$	$4\overline{)56}$	$? \div 9 = 7$	$68 \div 4 = ?$
$? \times 7 = 63$	$48 \div 8 =$	$6\overline{)84}$	$3\overline{)\,?}$ 24
$8\overline{)\,?}$ 13	$? \div 4 = 12$	$72 \div 9 =$	$? \times 7 = 84$
$? \times 11 = 88$	$? \times 4 = 36$	$7\overline{)\,?}$ 8	$15 \times ? = 75$
$?\overline{)78}$ 13	$\frac{1}{4}$ of $72 =$	$\frac{1}{7}$ of $56 =$	$? \times 2 = 46$
$54 \times 2 =$	$5\overline{)455}$	$56 \div ? = 2$	$\frac{1}{3}$ of $456 =$
$42 \times 3 =$	$\frac{1}{2}$ of $86 =$	$12\overline{)108}$	$12 \times ? = 132$
$45 \div ? = 3$	$9\overline{)189}$	$13\overline{)\,?}$ 4	$72 \div ? = 6$
$\frac{1}{12} \times 72 =$	$\frac{1}{5} \times ? = 3$	$63 \div ? = 7$	$128 \div 4 =$
$72 \div 3 =$	$? \div 6 = 7$	$63 \times 2 = ?$	$9 \times ? = 99$

I. R. B. Test [2]. Review of Fractions and Decimals.

Add.

1. 356.75, 1264.24, 761.16, 2364.92, 178.46.

2. $12\frac{1}{2}$, $15\frac{1}{4}$, $62\frac{3}{4}$, $24\frac{5}{8}$, $72\frac{1}{8}$, $65\frac{3}{8}$.

Multiply.

3. 325.62 by 421.164.

4. $24\frac{1}{2}$ by $45\frac{3}{4}$.

Divide.

5. 675.27 by 32.16 to three decimals.

6. $75\frac{1}{2}$ by $25\frac{1}{6}$.

Copy the following form, multiply each of the numbers in the left-hand column by each of the numbers at the top of the form:

	10	100	1000	200	40
22.5					
.035					
4.27					
$5.12\frac{1}{2}$					
$1.114\frac{1}{4}$					
2.739					
$.000\frac{1}{2}$					
.0721					
.654					

I. R. B. Test [3]. Review of Decimals and Per Cents.

Change each of the following decimals to per cents.

1. .58. **2.** .05. **3.** $.05\frac{1}{2}$. **4.** .128. **5.** .3456.

Change each of the following to per cents:

6. 2.50 **7.** 5. **8.** $7.12\frac{1}{2}$. **9.** $8.75\frac{1}{2}$. **10.** $12.37\frac{1}{2}$.

Change each of the following per cents to decimals or mixed decimals.

11. $\frac{1}{4}\%$. **12.** $\frac{1}{5}\%$. **13.** 215%. **14.** 565%. **15.** $425\frac{1}{4}\%$.

I. R. B. Test [4]. Review in Finding Percentages.

1. 3.6% of $432.56.

2. 22.4% of 675.35.

3. .65% of $2,000,000.

4. .45% of 400,000.

5. $3.75\frac{3}{4}\%$ of $800,000.

6. $52.5\frac{1}{2}\%$ of $80,000.

I. R. B. Test [5]. Review Using Per Cents Larger than 100%.

1. 525% of 60.

2. 350% of 8.

3. 750% of 16.

4. 975% of 32.

5. 2000% of $2.

6. 1500% of 12.

I. R. B. Test [6]. Review Using Percentage to Find Base or Whole.

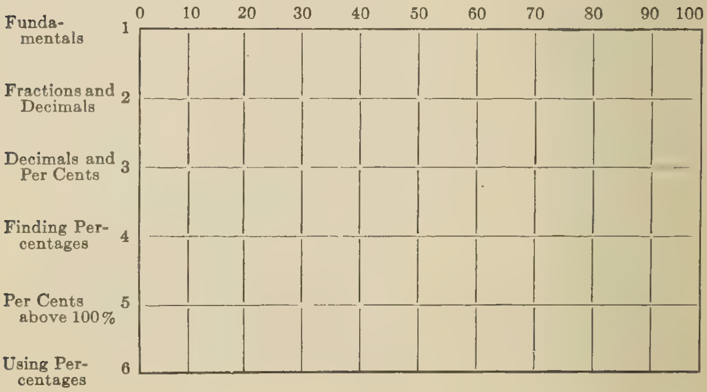
1. How many examples must Mary get right to have 80%, if she has 10 to work? 50? 90? 20?

2. How many times must a batter make hits if he is to have a batting average of 30% out of 60 trys? 80? 150? 240?

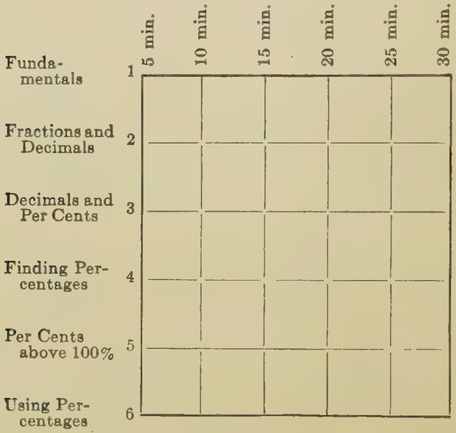
3. How many games must a team win to obtain 60% if they play 10 games? 25 games? 40 games? 125 games? 150 games?

4. How many sales must a salesman make if he is to sell 40% of his customers when he calls on 20? on 60 customers? on 15? on 75?

I. R. B. Test [7]. Graph of Accuracy Scores.



I. R. B. Test [8]. Graph of Time Records.



FOURTH TRIAL OF I. R. B. TESTS OF CHAPTER X

To complete the I. R. B. Chart of Chapter X turn back to its pages and take the Fourth Trial of the seven I. R. B. tests. Try faithfully to improve the first three records. After doing your best enter your new scores in the Fourth Trial column, even if they only equal the best previous record.

REVIEW AND FOURTH TRIALS OF I. R. B. TESTS OF CHAPTER XI

Before undertaking the following General Survey turn back to Chapter XI and take the three I. R. B. tests. Let the scores of your first attempt be the record of the Review Trial column. Then try for improved scores and enter these in the Fourth Trial column.

A General Survey. — To plan a way through a special field, men usually have a *survey*. This is an accurate view over the entire field. As we are about to make our way in review through the entire field of this book, it is well to start with a general survey.

FIRST GENERAL SURVEY WITH RELAY RACES

The Steering Committee and Teacher should divide the class into relay teams of four runners and one captain each, forming teams of as nearly equal abilities as possible. The captain should act as coach and scorekeeper. Each team should run the four races and each runner a different lap in each race. There are 80 examples in the following General Survey. You will note on the score card on the next page that there are 20 examples for each lap. By means of the four laps each runner will work the 80 examples.

The Steering Committee should act as referees to start and stop races. The committee should allow enough time for slow runners to make good accuracy scores but it must set a time limit to close one race for the start of the next. The captains will coach their teams to finish on time. Of course referees are privileged to penalize runners for any irregularity or the captains for giving too much assistance: these penalties will reduce a team's scores as the committee decides.

The captain should keep each runner's time and check the four answer papers to secure the accuracy scores, allowing five points for each of the 20 examples of each race. At the conclusion of the fourth race the captain should average each runner's time and accuracy scores and then find the averages for his team. The Steering Committee should determine the winning relay team.

Many other kinds of scores for individual runners and teams may be devised for the race. The resourcefulness of the captains and Steering Committee will add many interesting features to the above suggestions.

CAPTAIN'S SCORE CARD OF THE FIRST RACE

Team	Runners	Laps	Examples	Time	Accuracy Score
"Four-Color"	Red	1st	1–20		
	Blue	2nd	21–40		
	White	3rd	41–60		
	Green	4th	61–80		

Each captain should prepare four score cards like the above except that the order of the runners in the second, third, and fourth races would be:

Second Race			Third Race			Fourth Race		
Green	1st	1–20	White	1st	1–20	Blue	1st	1–20
Red	2nd	21–40	Green	2nd	21–40	White	2nd	21–40
Blue	3rd	41–60	Red	3rd	41–60	Green	3rd	41–60
White	4th	61–80	Blue	4th	61–80	Red	4th	61–80

First General Survey Review

1. Frank Jones sold at 2 for 5 cents some apples that he bought at 3 for 5 cents. There were 20 dozen apples. How many cents did he make? What per cent did he make on the cost of all the apples?

2. Add: $\frac{3}{4} + \frac{5}{8} + \frac{7}{12}$.

3. Multiply $3\frac{1}{2} \times \frac{6}{7} \times 3\frac{2}{3}$.

4. From the sum of .065, .00732, 1.76, and 2.861 subtract 1.47325.

5. A rectangular room 15 feet long and 12 feet wide is to have a rug 12 feet long and 9 feet wide. Find the area of the room; of the rug; of the margin.

6. Find the cost of painting a rectangular billboard 40 feet by 8 feet at 20 cents a square yard.

7. I loaned to H. C. Watson $185 on April 4, 1925. He paid me this debt with 6% interest on May 16, 1927. How much should he pay me?

8. Fred's mother bought a dress during a sale, which had been marked $15, but on which there were discounts of 15% and 10%. How much did she pay for it?

9. What is the ratio of 3 quarts to 5 pecks?

10. 12% of 150 is what per cent of 200?

11. What is the difference between 2 square yards and two yards square? Draw diagrams to scale to represent each.

12. What is a trapezoid? State the formula for the area of a trapezoid.

13. The formula $A = \pi r^2$ is a formula for the area of what?

14. What number (approximately) does π stand for in example 13?

15. I sold for $80 a cow that cost me $90. What was my loss per cent on the cost?

16. If you knew Mr. Brown's age, and you also knew how much older he is than his wife, how could you find Mrs. Brown's age?

17. A meter is equal to how many inches? (See appendix.)

18. Find the circumference of a wheel if its radius is $3\frac{1}{2}$ feet.

19. Write a promissory note for 6 months for $860.50, payable to Alfred Maguire with yourself as the maker, and bearing interest at 6%. Use the present date.

20. Compute the interest on Alfred's note the day it falls due. How much would pay the indebtedness in full?

21. Mr. Jones has a rectangular garden 165 feet long and 40 feet wide. This garden is set out to tomato plants, 6 to the square yard. How much did these plants cost at 75 cents a dozen?

22. If Mr. Jones averages $1\frac{1}{2}$ quarts of tomatoes from each of his plants, how many bushels will he have?

23. Find out the number of cases of tardiness in your school for the past month. Draw a bar graph to represent these facts by grades.

24. How far is it across the widest part of a circular lake which is 1 mile around?

25. Construct a triangle whose sides are $1\frac{1}{4}$ inches, $1\frac{1}{4}$ inches, and 1 inch. Give the names that apply to this triangle.

26. The vestibule in Mr. Washburn's house is 6 feet long and 4 feet wide. It is covered with tiles one inch square. Find the cost of these tiles at 4 cents each.

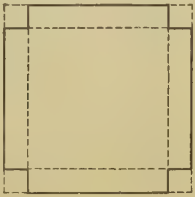
27. Mr. William's lot is a square 80 by 80 feet. Mr. Henry's is a rectangle 160 feet by 40 feet. Which is the larger lot? Which will require the longer fence to inclose it? How much longer?

28. The state built a board fence a mile long along a state road. The posts were set 12 feet apart. How many posts were required? What was their cost at $1.75 each?

A STATE ROAD WITH POSTS

This shows part of the beautiful Columbia River Highway in Oregon.

29. Mrs. Saxer bought a coat at a discount of 15% and paid $42.50 for it. What was the marked price of the coat?



30. Ten per cent of ten per cent is what per cent?

31. A square piece of tin 15 inches on a side had squares three inches on a side cut from each corner as shown. The tin was then bent up along the dotted lines, thus forming an open box. How many square inches of tin in the bottom of the box? in the whole box? How deep is the box?

32. Reduce 3 miles 4 yards to feet.

33. How many acres in 840 square rods?

34. Change 50,000 millimeters to meters; to inches; to feet.

35. How many square hectometers in 40,000 square meters? How many ares?

36. Reduce 4 hogsheads, 1 barrel, 15 gallons to quarts.

37. How many bushels in 1280 pints?

38. Change 175 days to weeks.

39. $14\frac{1}{2}$ gross of pencils is how many pencils?

40. 7850 pounds of coal is how many tons?

41. 15 kilometers is how many miles?

42. How many liters in 105.67 quarts?

43. A tank has a capacity of 1000 cubic feet. How many gallons will it hold? If the tank were full of water, how many pounds would the water weigh? How many kilograms?

44. I received $2880 for a lot after a real estate agent had deducted 4% commission for selling it. For how much was the lot sold?

45. A man offered a certain article for $1200 with discounts of 25% and $16\frac{2}{3}$% or a discount of 30% and 10%. Which offer is the better and how much better?

46. A boy rode his bicycle $26\frac{1}{4}$ miles in $3\frac{1}{2}$ hours. What was his average rate per hour?

47. I sold a used automobile for $960. If this was 75% of what it cost when new, what did it cost when new?

48. What is the interest on $672 at $5\frac{1}{2}$% for 2 yr. 1 mo. 15 da.?

49. By selling a piece of property for $25,000 I lose $5000. What per cent do I lose on the cost?

50. In a class in Junior Mathematics 4 failed, 6 were E (90% or better), 10 were G (80% to 90%), and 16 were fair (70% to 80%). Construct bar graphs to represent these numbers. Find what per cent of the class are in each group.

51. The diameter of an automobile wheel is 36 inches. Find the circumference. How many revolutions would the wheel make in going 10 miles?

52. A class was given the following example: $66\frac{2}{3}\%$ of \$1500 = ? John got for an answer, \$100; Henry \$2000; and George \$1200. Estimate which was nearest correct before you actually work it. Now work it. What is your result?

53. A man pays \$32 for a railroad ticket which is to take him 1000 miles. If he travels 125 miles on R.R. A, 200 miles on R.R. B, 175 miles on C, and 500 miles on D, how much should each R.R. receive?

54. If the taxes, insurance, and repairs on a house, which cost \$5000, are \$200 a year and the owner rents the house for \$50 a month, what net profit per cent does the owner make on the cost of the house for a year?

55. If a rectangular hall 10 yards wide contains 1500 square feet of floor space, how long is it?

56. A diamond merchant gained \$120 on the sale of a diamond ring. If he gained 25% of the cost to him, what did it cost him?

57. Mr. Strader marked a damaged chair at 30% below cost. If the marked price was \$28, what did it cost him?

58. A merchant who failed in business was able to pay 72% of his debts. How much could he pay Mr. Gladstone if he owed Mr. Gladstone \$2400?

59. A man's income is \$2500 a year. His expenses are: rent \$500, heat \$100, light \$30, household \$700, clothing \$300, miscellaneous \$200. What per cent of his salary does he save?

THE FLORIST SHOP

60. A florist bought 36 roses for $7.20. He made them up into 3 bouquets of 1 doz. each. He then sold the bouquets at $3 each. What per cent did he make on the cost?

61. An iceman pays 50 cents for each 200 pounds of ice. He sells it in 25-cent pieces. What per cent profit does he make on the cost if he cuts 7 pieces out of each 200 pounds?

62. If the upper limit of expenses of a family are: food 25% of income, rent 20%, fuel and light 15%, clothing 15%, car fare 10%, savings, doctor, pleasure, and miscellaneous 15%, find the amounts that would be apportioned on the following incomes: $2500. $3500.

If the food, rent, and fuel should each increase in cost 20% in any given year, how much would each of the above have to have their incomes increased to meet the increased expenses?

63. A study was made of a group of boys and girls who left school at the completion of their grammar school course.

Another study was made of a group who left school at the completion of their high school course. The following were the results:

AGE	AVERAGE WAGES OF THOSE LEAVING SCHOOL AT 14	AVERAGE WAGES OF THOSE LEAVING SCHOOL AT 18
14–16	$200 per year	In school
16–18	$250 per year	In school
18–20	$350 per year	$500 per year
20–22	$475 per year	$750 per year
22–24	$575 per year	$1050 per year
24–25	$600 per year	$1150 per year
25–26	$688 per year	$1550 per year

What is the total amount received by the first group?

What is the average wage per year of the first group?

What is the total amount earned by the second group?

What is the average wage per year of the second group?

What is the difference of the total amounts of the two groups?

If there were 190 days in each school year, how much was each day spent in high school worth to each one of that group?

64. Find the perimeter of a triangle if the sides are 8 feet 6 inches, $2\frac{1}{4}$ yards, and 6 feet 9 inches.

65. A 9-foot by 12-foot rug was advertised at $95. How much was that per square yard to the nearest cent?

66. The perimeter of a rectangle is 180 rods. If the length is 55 rods, what is the width? How many acres does it contain?

67. A wholesaler bought a stock of merchandise for $16,000. He sold it at a gain of 25% on the cost but was able to collect only 25% of his bills. How much did he make or lose on the transaction? What per cent was this on the cost to him?

68. What is the interest on $2800 for 10 months 15 days at 6%?

69. Find the saving and the per cent of saving on each of the following:

	STORE SELLING ON CREDIT	CASH STORE	SAVING	PER CENT OF SAVING
Groceries .	$25	$23.75		
Clothing . .	40	36.		
Shoes . . .	8	7.50		
Hats . . .	5	4.50		

70. A boy bought a catcher's mitt for $4.50. If he sent the pay by postal money order, what was the total cost to him?

71. A tailor sold a misfit suit for $34.00 at a loss of 15% of the cost to him. What did it cost him?

72. A man gained 200% on a certain article which he sold for 10 cents more than it cost him. What did it cost him?

73. An article costing $30 is marked to sell at $48, but is sold for $44. What per cent is made on the cost?

74. How much less is $\frac{3}{4}$ of $64 than $\frac{3}{4}$% of $12,000?

75. 56.34% of $5764 = ?

76. A man sold a house for $6400 at a loss of 20% on the cost. How much would he have gained had he sold it for $8500? What per cent on the cost?

77. How many meters in 2 miles if 1 meter = 3.28 feet (approximately)?

78. If a 1 horse power electric motor consumes 746 watts of electric energy per hour, how much will it cost to run it on an average of 6 hours a day for 10 days in a month at 10 cents per kilowatt-hour?

79. About 18 square feet of floor area should be allowed for each pupil in a room. Measure your floor and find out how many pupils should be in your room.

80. $\frac{1}{2}$ is what per cent of $\frac{3}{4}$?

SECOND SURVEY — TIMED TESTS

A New Method of Scoring, by "Medians"

In the following 17 Timed Tests why not agree with your classmates to take them in out-of-school-hours? Possibly you may decide to work in committees at some time in school or in some one's home. Thus in this last test you will be taking just as you have heretofore not only a test in mathematics but also the more important honor test.

A new method of scoring by *medians* will give you a final new feature to add to your I. R. B. Show your resourcefulness by originating your own plan of entering your time and accuracy scores and the *median* score of your class.

On the day following your work on the Timed Tests have the pupils stand in the order of the best time scores until half the class is standing. The time score of the last pupil to stand in this half of the class will be the *median* time of the whole class. In a similar way have pupils stand in the order of the best accuracy scores until half the class is standing. The accuracy score of the last pupil to stand in this half of the class will be the *median* accuracy score of the whole class.

Before you start on this Second Survey recall some of the standards which you have set up for worthy Junior High School girls and boys, standards which hold for all good workmen: skill; accuracy; speed of work; honor; resourcefulness; and 100% trustworthiness.

Now in these last 17 Timed Tests prove to yourself that you have done your best to measure up to these standards; and when you are all through still keep right after your highest standard of all: IMPROVEMENT.

Second Survey — Timed Tests

You should practice on each of the following timed tests until you can complete each of them in the allotted time. The time allotment for tests **1** to **4** is 2 minutes, with not more than one error. Do not copy the examples.

1.

Add:

6	5	7	8	0	3	5	9	4	5	6	3
3	2	9	6	7	7	6	3	8	9	1	0

8	3	6	4	6	2	9	8	5	6	3	9
4	9	5	7	4	8	7	6	9	2	9	9

2.

Subtract:

7	8	7	9	10	15	16	13	12	11	16	18
2	5	3	5	6	8	6	7	9	3	9	9

9	17	15	16	13	11	10	17	13	12	13	16
6	9	7	7	9	7	8	8	8	7	4	8

3.

Multiply:

5	6	5	8	9	6	9	8	7	0	1	9
3	2	7	6	3	7	2	7	8	9	7	7

7	8	9	7	4	3	6	4	3	2	8	7
7	4	4	6	9	7	4	9	6	9	0	4

4.

Divide:

12 ÷ 3 =,
16 ÷ 2 =,
36 ÷ 4 =,
72 ÷ 8 =,
48 ÷ 6 =,
27 ÷ 3 =,
64 ÷ 8 =,
48 ÷ 8 =,
42 ÷ 6 =,
36 ÷ 6 =,
28 ÷ 7 =,
16 ÷ 8 =,
27 ÷ 9 =,
63 ÷ 7 =,
72 ÷ 9 =,
54 ÷ 9 =,
15 ÷ 3 =,
18 ÷ 6 =,
21 ÷ 7 =,
32 ÷ 4 =,
42 ÷ 7 =,

5.

Add as many of the following, without copying, as you can in 6 minutes. Be sure to keep a record of how many you attempt and how many (or what per cent) of those that you attempt you get right.

1.	*2.*	*3.*	*4.*	*5.*	*6.*
576	457	125	918	236	327
342	721	342	624	642	452
621	162	216	512	215	125
147	417	714	264	426	462
382	238	283	190	901	910
314	143	134	473	347	374
568	865	856	476	467	647

7.	*8.*	*9.*	*10.*	*11.*	*12.*
508	805	850	624	246	426
782	278	267	672	762	458
472	427	724	742	481	184
629	692	296	269	345	354
826	862	268	286	567	576
148	418	184	481	369	396
532	523	325	352	362	326

13.	*14.*	*15.*	*16.*	*17.*	*18.*
361	316	631	613	472	247
672	267	276	462	436	520
372	328	284	248	531	421
641	142	321	324	521	321
725	752	257	836	732	237
842	853	358	842	521	825
175	174	189	173	184	285

6.

4 minutes for each of the next three tests.

Subtract:	4321	9673	7708	2176	8473	7277
	1087	8421	6397	1091	5169	5208
	3160	8432	4320	6030	9172	8593
	2941	8022	1877	4377	4621	4287

7.

Multiply:	725	864	923	576	329	765
	25	32	36	40	80	36
	925	846	923	864	725	150
	20	15	12	70	60	12

8.

Divide:

18)144	16)256	14)182	25)625	30)750	42)840
19)361	18)270	16)128	32)512	18)324	18)540

9.

Time: 5 minutes.

Add: $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + 2 + 4\frac{1}{2} + 6\frac{1}{3} + 3\frac{2}{3}$.
$\frac{3}{5} + \frac{5}{8} + \frac{3}{4} + 1\frac{4}{5} + 2\frac{1}{5} + 3\frac{1}{4} + 6$.
$\frac{3}{4} + \frac{1}{2} + \frac{3}{2} + 2\frac{1}{3} + \frac{5}{6} + \frac{2}{3} + 2$.
$4\frac{1}{2} + 3\frac{2}{3} + 4\frac{1}{8} + 5\frac{1}{3} + 6\frac{3}{4} + 2\frac{1}{2} + 4\frac{1}{4}$.

10.

Time: 4 minutes.

.0685 + .0532 + .7549 + .6532 + .8263.
.758 + .07932 + .8643 + 7.25 + 6.052 + 16.01.
3 ft. 4 in. + 8 ft. 6 in. + 10 ft. 3 in. + 7 ft. 6 in.
10 yr. 3 mo. 8 da. + 4 yr. 8 mo. 12 da. + 3 yr. 11 mo. 24 da.

11.

Time: 4 minutes.

(*a*) Add and check by adding the opposite way; that is, first up and then down:

\$28.56 42.91	\$37.44 63.40	\$108.72 59.24	\$11.86 29.77	\$175.42 36.91
\$623.84 256.17	\$893.77 472.81	\$311.84 476.29	\$330.18 117.27	\$301.16 976.03

(*b*) Subtract and check by adding the remainder to the subtrahend:

\$377.06 192.25	\$376.21 178.55	\$873.12 479.83	\$645.17 377.52	\$843.06 448.75
\$573.21 146.03	\$584.66 329.75	\$574.07 362.95	\$942.05 475.75	\$3274.16 1176.92

12.

Time: 5 minutes.

(*a*) *Multiply:*

\$186.74 25	\$732.10 36	\$147.25 60	\$175.20 45	\$256.50 80
	\$1312.75 15	\$1256.20 35		

(*b*) *Divide:*

22)\$383.90 35)\$916.30 24)\$380.40 18)\$661.50 45)981.00

13.

Time: 2 minutes.

Find:

20% of 175.
$16\frac{2}{3}$% of 96
$33\frac{1}{3}$% of 162.
$8\frac{1}{3}$% of 12 doz.
75% of 96 in.

$37\frac{1}{2}$% of \$128.
$87\frac{1}{2}$% of 64.
$83\frac{1}{3}$% of 54 bu.
40% of 175 men.
80% of 72 cents.

14.

Time: 10 minutes.

On a blank sheet of paper draw three columns similar to the following, copy the headings, and copy the per cent column. Close your book and on your own paper fill in the Decimal and Fraction columns. Then open the book and check your work. Continue until you can make a 100% record.

PER CENT	DECIMAL	FRACTION
$12\frac{1}{2}$	$.12 5$	$\frac{1}{8}$
$33\frac{1}{3}$	$.33\frac{1}{3}$	$\frac{1}{3}$
25	.25	$\frac{1}{4}$
40	.40	$\frac{2}{5}$
75	.75	$\frac{3}{4}$
$66\frac{2}{3}$	$.66\frac{2}{3}$	$\frac{2}{3}$
50	.50	$\frac{1}{2}$
10	.10	$\frac{1}{10}$
80	.80	$\frac{4}{5}$
$87\frac{1}{2}$	.875	$\frac{7}{8}$

PER CENT	DECIMAL	FRACTION
$62\frac{1}{2}$	.625	$\frac{5}{8}$
$16\frac{2}{3}$	$.16\frac{2}{3}$	$\frac{1}{6}$
60	.60	$\frac{3}{5}$
70	.70	$\frac{7}{10}$
$6\frac{1}{4}$	.0625	$\frac{1}{16}$
$8\frac{1}{3}$	$.08\frac{1}{3}$	$\frac{1}{12}$
20	.20	$\frac{1}{5}$
$37\frac{1}{2}$	.375	$\frac{3}{8}$
$83\frac{1}{3}$	$.83\frac{1}{3}$	$\frac{5}{6}$
30	.30	$\frac{3}{10}$

15.

Time: 4 minutes.

The following examples have the correct answers written at the top of the next page. See if you can estimate them and fit the proper answer to each example.

1. $216\frac{1}{2} + 674\frac{2}{3} + 356\frac{3}{8} + 125\frac{5}{8} =$

2. $3512 + 4673 + 8539 + 5643 + 9325 + 7768 =$

3. $76 \times 565 =$

4. $\frac{2}{3} - \frac{3}{8} =$

5. $25\frac{1}{3} + 36\frac{1}{2} + 72\frac{3}{4} + 84\frac{5}{8} =$

6. $763.2 \div 1.8 =$

7. $965.4 + 324.6 + 142.3 + 356.4 + 124.8 =$

8. $\frac{3}{4} - \frac{5}{16} =$

Answers: 424; $1373\frac{1}{6}$; 1913.5; 39460; $219\frac{5}{24}$; $\frac{7}{16}$; 42940; $\frac{7}{24}$.

16.

Time: 2 minutes.

Divide and check:

1. $\frac{5}{6} \div 5$.
2. $\frac{2}{3} \div 6$.
3. $\frac{7}{9} \div 3$.
4. $\frac{4}{5} \div 2$.
5. $\frac{5}{8} \div 4$.
6. $\frac{9}{16} \div 9$.
7. $\frac{1}{2} \div \frac{1}{4}$.
8. $\frac{2}{3} \div \frac{3}{4}$.
9. $\frac{5}{9} \div \frac{6}{7}$.
10. $5\frac{1}{2} \div 11$.
11. $8\frac{2}{5} \div 7$.
12. $9\frac{5}{8} \div 5\frac{1}{2}$.

17.

Time: 3 minutes.

Complete the following:

3 dimes is —— per cent of 90 cents.
1 peck is —— per cent of 1 bushel.
1 pint is —— per cent of 1 gallon.
6 inches is —— per cent of 1 yard.
5 cm. is —— per cent of 1 meter.
15% of 2 dimes is —— cents.
35% of 3 bushels and 4 quarts is —— quarts.
200% of 3 dimes is —— per cent of a dollar.
25% of 1 mile is —— yards.
8 feet is $16\frac{2}{3}$% of —— yards.

When you have completed the independent work in the timed tests, determine with the help of your Steering Committee which tests should be reviewed in class period time; that is, which tests have given general difficulty to all the class. Then take these tests one by one in the classroom.

END-OF-CHAPTER, REVIEW, AND FOURTH TRIALS OF I. R. B. TESTS, CHAPTER XII

Before you close this last chapter you will want to improve your First Trial scores in its six I. R. B. tests. Try them at least three times and enter your three scores in the End-of-Chapter, Review, and Fourth Trial columns. Try faithfully to surpass your first scores and to equal or excel the class average scores. Draw graphs to represent your best time and accuracy scores of all four attempts.

Third Survey. A Good Fun Review

By all means work these problems together. Let the Chairman of your Steering Committee take charge of the class period. Have all the fun you can. A sense of humor is a good quality to develop. It will help you to get along with folks and getting along with folks is always worth a great deal. An employer, a manager, or any one who directs the work of other people will get along better with his people if he possesses a sense of humor.

When you have finished the exercise see who in your class can recall some catch problem which he has heard before. There are some tricks with numbers which make good recreations. Have your classmates inquire among their friends to find other " recreations " in mathematics.

Perhaps you may want to place in your Improvement Book what appealed to you as the best " recreations." By writing them you will help yourself to remember them better. There will be many occasions when these recreations will help you to get along with folks.

Recreations

1. A bottle and a cork cost $1.10. The bottle cost $1 more than the cork. What was the cost of each?

2. A frog is in the bottom of a 30-foot well. If he jumps up 3 feet each day and falls back 2 feet each night, how many days will it take him to get out of the well?

3. | | | | | |. Add 5 strokes and make 9.

4. How would you write 11 thousand, 11 hundred, 11?

5. There is a room with 8 corners. In every corner there is a cat; in front of every cat there are 7 cats, and on every cat's tail there is a cat. How many cats are there?

6. If an orange and a half costs a cent and a half, what will 10 oranges cost?

7. Write 100 with 4 figures, using no zeros.

8. State a word of five letters from which if you take two away you will have one.

9. Suppose that a train were to leave New York at 4 minutes past eight in the morning toward Philadelphia. Suppose that at eight minutes past nine the same morning a train leaves Philadelphia for New York. When they passed which train would be nearer to New York?

10. Assuming that the train for Philadelphia were making an average speed of 45 miles per hour and the other train made an average of 50 miles an hour, how fast would a man be moving who stood between them when they passed?

11. The New York bound train will arrive at its destination at 11.08 and the other will arrive at Philadelphia at 10.52. What time will it be when the engineer's watch on the faster train says 9.37?

12. A man sold a horse for $90, bought him back for $80, and resold him for $100. What did he make on the transaction?

13. A boy driving some ducks home was asked how many he had. He replied "When in line, there are two ducks ahead of a duck, two ducks behind a duck, and a duck in the middle." How many ducks were there?

14. Answer each of the following questions:

How many units in 12?
How many two's in 12?
How many three's in 12?
How many four's in 12?
How many sixes in 12?

How many eggs in a dozen?
How many 1¢ stamps in a dozen?
How many 2¢ stamps in a dozen?
How many 3¢ stamps in a dozen?

15. Place 8 tooth picks or matches in a row. Jump two at a time and have four piles of two each, allowing no pick to jump twice.

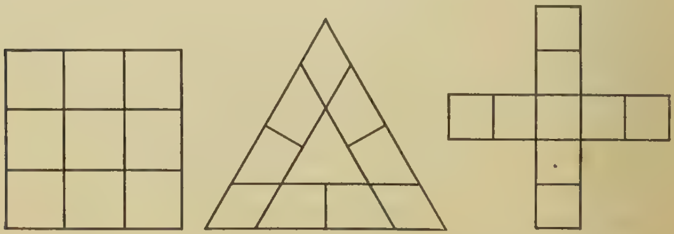
16. Which is greater, six dozen dozen or a half dozen dozen?

17. A man bought a pair of shoes for $5 and gave the shoe dealer a $10 bill in payment. The dealer could not change the bill so he asked a neighboring merchant to change the bill and then he gave the man the change. The neighboring merchant came back in a short time and said that the bill was a counterfeit, so the shoe dealer gave him a good bill for it. What was the shoe dealer's loss?

18. What word begins with a *y* and ends with a *y*, has its first syllable a word and its last syllable a word?

19. $6 - 6 \div 6 + 6 \times 2 - 2 = ?$

20. Arrange the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, in each of the following forms, using no figure twice in each figure, so that the sum any way in the square is 15, in the triangle is 20, and the cross is 27.



APPENDIX

ADDITIONAL WORK IN GEOMETRY

Intersections. — Did you ever drive along a road near a railroad track, and find yourself crossing that same track again and again? This happens because usually neither the railroad track nor the highway are straight. If both were straight, how many crossings could there be? *Two straight lines can intersect (cross) in only one point.* In how many points can a straight line intersect a circle? In how many points can two circles intersect each other?

Circles and Lines. — In Figure 1, name the line *OA*; the line *BA*; the straight line *CD*. (See page 81.) What is the curved line *CD* called?

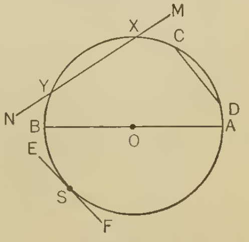


Figure 1

The line *M N* of indefinite length, cutting the circle in the points *X* and *Y*, is called a *secant*. How does a secant differ from a chord? The line *FE* just touches the circle in one point, *S*, that is to say, *S* is the only point which is on the line and also on the circle. This line is called a *tangent*, and the point *S* is called the *point of contact.*

Circles and Angles. — In Figure 2, the angle *AOB* has its vertex at the center of the circle and its sides are radii. Such an angle is called a *central angle.* The arc *AB* is said to be *intercepted* by this angle. Similarly the angle *COA* is a central angle equal to the angle

AOB. Its arc *AC* is equal to the arc *AB*. *In the same circle, equal central angles intercept* (*or have*) *equal arcs.* Likewise the angle *BOD* is a central angle, and *BD* is its intercepted arc.

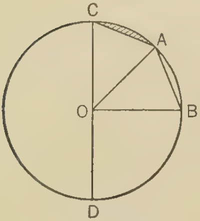


Figure 2

If the angle *BOD* is twice the angle *BOA*, then arc *BD* is twice arc *BA*. If angle *BOD* is a right angle (an angle of 90 degrees), then the arc *BD* is an arc of 90 degrees. Arcs, like angles, are measured in degrees. An angle degree is $\frac{1}{90}$ of a right angle, while an arc degree is $\frac{1}{360}$ of a circle. Because of this relation the number of angle degrees in a central angle is the same as the number of arc degrees in its intercepted arc. Hence *a central angle is measured by its intercepted arc.*

Since the arcs *AC* and *AB* are equal, the chords *AC* and *AB* are also equal, for *in the same circle, equal arcs have equal chords.* If arc *BD* is twice arc *AC*, is chord *BD* twice the chord *AC*? Verify your statement by measurement. Can you explain why this is true?

Circles and Areas. — A figure bounded by two radii and their intercepted arc such as *DOB* is called a *sector.* A sector which equals one fourth of a circle is called a *quadrant.* Point out a quadrant in the figure. The area bounded by a chord and its arc, like the shaded area in the diagram, is called a *segment of a circle.*

Exercises

1. Draw a circle with a diameter of 3 inches. Then draw one of its radii. Construct a line perpendicular to this radius at its outer end. Name the line you have constructed.

2. Draw any tangent to another 3-inch circle. Draw the radius to the point of contact. What kind of angle is formed? Verify by measuring the angles with your protractor.

3. Draw a third circle and construct two tangents to the circle from the same external point. Measure the length of each tangent from that point to the point of contact. What do you conclude?

4. Draw two circles which do not intersect or touch. How many *common tangents* can you draw to these circles?

5. Try drawing two circles in such positions that only three common tangents can be drawn. Only two. Only one. None.

6. That part of a secant included within a circle may be called a ——.

7. Would the following statements be true if they read "equal circles" instead of "the same circle"?

(*a*) In the same circle, equal central angles intercept equal arcs.

(*b*) In the same circle, equal arcs have equal chords.

8. In Figure 3, AOB is a diameter. Angle BOC is an angle of 35 degrees. How many degrees are in the angle AOC? in the arc BC? in the arc AC?

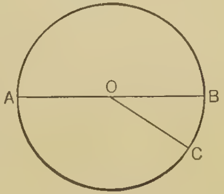


Figure 3

9. In Figure 4, AOC is an angle of 60 degrees. What part of the circle is it? If the radius of the circle were 7 inches, what would be the area of the circle? Find the area of the sector.

10. In Figure 4, what kind of a triangle is the triangle AOC? Measure its altitude and compute its area. Now how can you find the area of the segment, S?

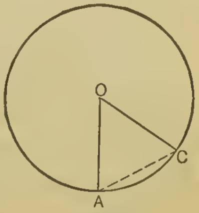


Figure 4

11. Draw a circle with a two-inch radius. Construct two perpendicular diameters. Join the ends of these diameters. What kind of figure is formed? Can you find its area? Subtract its area from that of the circle, and divide by 4. Now point out a segment whose area you have just found.

12. Draw a circle and at its center construct, in order, angles of 40 degrees, 60 degrees, 70 degrees, 30 degrees, 90 degrees, and 20 degrees. How many degrees in the remaining angle? How many degrees in each of the intercepted arcs?

13. *Parallel lines intercept equal arcs on a circle.* Draw a figure to illustrate the truth of this statement (*a*) when the parallel lines are secants, (*b*) when they are tangents, (*c*) when one is a secant and the other a tangent.

14. A *ray* is a straight line beginning at some given point and extending indefinitely in one direction from it. Draw four rays extending in different directions from some point P.

15. A *line segment* is the part of a straight line between any two points (page 77). Draw a straight line six inches long. Mark off on it four line segments of one inch each. Measure them with your compasses and see how accurate you were.

16. Why does it make for inaccuracies to use the ends of a ruler in measuring?

Why might three people get a slight difference in results when measuring the same line segment?

17. Can more than two lines intersect at the same point? If so, how many?

Areas of Irregular Figures. — We have learned how to find the area of a triangle, a square, a rectangle, a parallelogram, and a trapezoid (pages 132 to 145). If an irregular figure is bounded by straight lines, we can usually divide it by drawing other lines into figures of the types named above, and thus find its area. Study

Figures 5 to 9, and explain how the area can be found in each case.

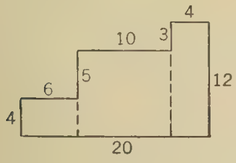


Figure 5

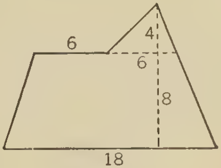


Figure 6

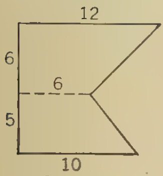


Figure 7

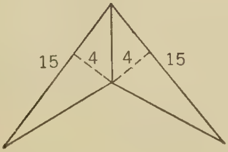


Figure 8

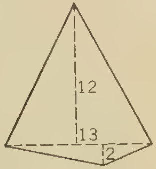


Figure 9

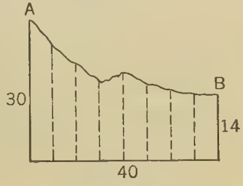


Figure 10

If one or more sides are not straight lines, the following example illustrates a method which can be used. Figure 10 represents a field bounded on one side, *AB*, by a creek. The dimensions represent rods. Divide the length of the field into eight intervals of 5 rods each. At each of these points erect perpendiculars as shown. Measure the lengths of these perpendiculars. Taking them in order from left to right, suppose them to be 25, 22, 20, 21, 18, 16, and 15 rods. Now add 30 and 14 and twice the length of each of the seven perpendiculars. Multiply the sum by two and one-half, which is half the distance between the perpendiculars. This will give the approximate area. What result do you obtain? See if you can explain the above rule. Notice that the field is divided into figures which are approximate trapezoids, each with an altitude of five rods. What is the rule for finding the area of a trapezoid? Express this rule as a formula.

Compute the area of each of the above figures with the dimensions shown.

TABLES YOU SHOULD KNOW

Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$272\frac{1}{4}$ square feet or $30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile
1 acre = 43,560 square feet

Metric Linear Measure

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 meter (m.)
10 meters = 1 decameter (dm.)
10 decameters = 1 hectometer (hm.)
10 hectometers = 1 kilometer (km.)
10 kilometers = 1 myriameter (mm.)

Metric Area

100 sq. millimeters (sq. mm.) = 1 square centimeter (sq. cm.)
100 sq. centimeter = 1 square decimeter (sq. dm.)
100 sq. decimeter = 1 square meter (sq. m.)
100 sq. meters = 1 square decameter (sq. dm.)
100 sq. decameters = 1 square hectometer (sq. hm.)
100 sq. hectometers = 1 square kilometer (km.)
1 sq. decameter is also called an are.
1 sq. hectometer is also called a hectare.

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 business month (for interest work, etc.)
12 months = 1 year (yr.)
365 days = 1 year
366 days = 1 leap year
10 years = 1 decade
100 years = 1 century

Measures of Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
90 degrees = 1 right angle
360 degrees of arc = 1 circumference
360 degrees of angle = 1 complete rotation

Counting

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gr.)
12 gross = 1 great gross (gt. gr.)
20 units = 1 score
144 units = 1 gross

Common Weight

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight or 2000 pounds = 1 ton (T.)

Paper Measure

24 sheets = 1 quire
500 sheets = 1 ream

Metric Liquid

10 milliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 liter (l.)
1 liter is a little more than 1 quart.

Metric Weight

10 millograms = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 gram (g.)
1 cu. cm. of water weighs 1 gram.

Important Equivalents

1 liter = 1.0567 quarts (exact)
1 meter = 39.37 inches
1 kilometer = .621+mile.
1 kilogram = 2.2 pounds.
1 gallon = 231 cubic inches.
1 bushel = 2150.42 cu. in.
1 ton of hard coal fills 35 cubic feet of space (approximately).
1 cubic foot holds 7½ gallons (approx.).
1 cu. ft. of water weighs 62.321 lbs.
60 pounds of wheat, potatoes, and beans to the bushel
32 pounds of oats to 1 bushel.
1 nickel weighs about 5 grams.
1 kilometer is ⅝ of a mile (approx.).
1 decimeter is 4 inches (approx.).

Measures for Recipes

3 teaspoonfuls (tsp.) = 1 tablespoonful (tbsp.)
2 gills = 1 cup
2 cups of granulated sugar = 1 pound
2 cups butter = 1 pound
1 pint = 1 pound

United States Money

10 mills (mi.) = 1 cent (¢) or (ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 Eagle

English Money

4 farthings (far.) = 1 penny (d.)
12 pence = 1 shilling (s.)
20 shillings = 1 pound or sovereign (£)
1 English pound = $4.8665

French Money

10 centimes (ct.) = 1 decime (dc.)
10 decimes = 1 franc (fr.)

1 franc was worth 19.3 cents before the World War. Ask your bank for its present value.

German Money

100 pfennige = 1 mark

1 mark was worth 23.8 cents before the war. Ask your banker its value now.

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